

ND-A182 927

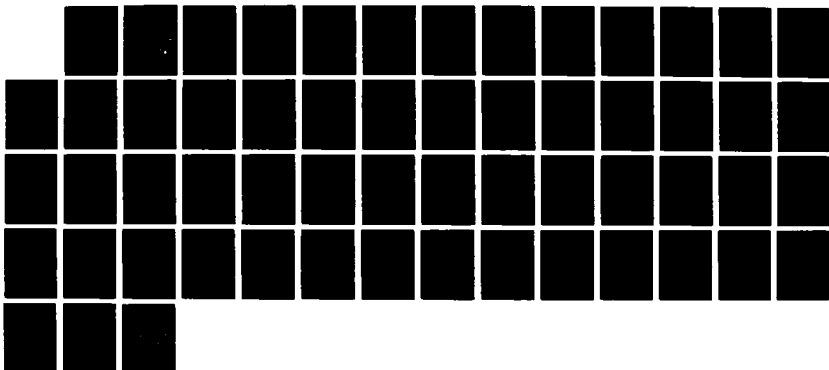
ANALYSIS OF THE SPECTRA OF TRIPLY IONIZED IRON IN  
RARE-EARTH ALUMINUM GARNETS(U) HARRY DIAMOND LABS  
ADELPHI MD C A MORRISON ET AL. JUL 87 HDL-TR-2113

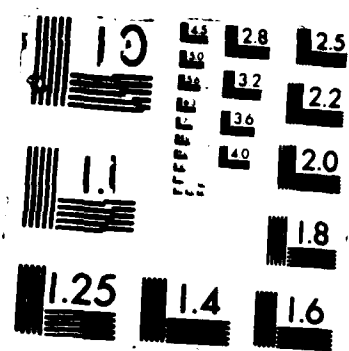
1/1

UNCLASSIFIED

F/G 7/2

NL





12

DTIC FILE COPY

HDL-TR-2113

July 1987

**Analysis of the Spectra of Triply Ionized Iron  
in Rare-Earth Aluminum Garnets**

by Clyde A. Morrison  
John D. Bruno  
Gregory A. Turner

DTIC  
ELECTE  
JUL 3 1 1987  
S D  
C E

AD-A182 927



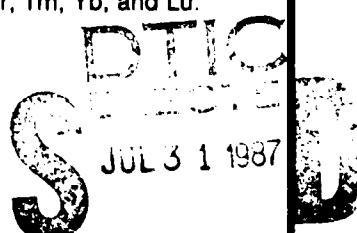
**U.S. Army Laboratory Command  
Harry Diamond Laboratories  
Adelphi, MD 20783-1197**

Approved for public release; distribution unlimited.

37 7 29 028

A152 927

# REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT  Approved for public release; distribution unlimited.		
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S)  HDL-TR-2113			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION  Harry Diamond Laboratories		6b. OFFICE SYMBOL (If applicable)  SLCHD-RT-RA	7a. NAME OF MONITORING ORGANIZATION		
6c. ADDRESS (City, State, and ZIP Code)  2800 Powder Mill Road Adelphi, MD 20783-1197			7b. ADDRESS (City, State, and ZIP Code)		
8a. NAME OF FUNDING / SPONSORING ORGANIZATION  Center for Night Vision and Electro-Optics		8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c. ADDRESS (City, State, and ZIP Code)  Ft Belvoir, VA 22060			10. SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO.  61102A	PROJECT NO.	TASK NO.
					WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification)  Analysis of the Spectra of Triply Ionized Iron in Rare-Earth Aluminum Garnets					
12. PERSONAL AUTHOR(S)  Clyde A. Morrison, John D. Bruno, and Gregory A. Turner					
13a. TYPE OF REPORT  Interim		13b. TIME COVERED FROM Jun 86 TO Sep 86		14. DATE OF REPORT (Year, Month, Day) July 1987	
15. PAGE COUNT 56					
16. SUPPLEMENTARY NOTATION  AMS code: 611102.31B0011; HDL project: 590751; PRON: CJ727269-01-CJ-CT					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Rare-earth aluminum garnets, optical spectra of Fe <sup>3+</sup> , transition metal ion spectra, crystal-field splittings in garnets, crystal field theory		
20	02				
20	05				
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  Experimental data on the spectra of Fe <sup>3+</sup> in seven rare-earth aluminum garnets are analyzed. The first analysis consists of the conventional cubic approximations to obtain the Slater parameters F <sup>(2)</sup> , F <sup>(4)</sup> and the crystal field parameters, B <sub>40</sub> . The concept of rotational invariance of the crystal fields, along with the point charge lattice sums, was used to predict the crystal field parameters for the Al <sub>2</sub> site with C <sub>3i</sub> symmetry. The best fit Slater parameters, along with the spin-orbit parameter, were used to predict the optical spectra of Fe <sup>3+</sup> in R <sub>3</sub> Al <sub>5</sub> O <sub>12</sub> for R = Gd, Tb, Dy Ho, Er, Tm, Yb, and Lu.					
<div style="text-align: right;">  </div>					
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		
22a. NAME OF RESPONSIBLE INDIVIDUAL  Clyde A. Morrison			22b. TELEPHONE (Include Area Code) (202) 394-2042		22c. OFFICE SYMBOL SLCHD-RT-RA

## CONTENTS

	Page
1. INTRODUCTION .....	5
2. THEORY .....	5
3. WAVE FUNCTIONS AND FITTING THE EXPERIMENTAL DATA .....	7
3.1 Cubic Approximation .....	8
3.2 Theoretical Crystal Field Parameters .....	16
3.3 Final Parameters and Predicted Splittings, Including Spin-Orbit Interaction .....	17
4. DISCUSSION, CONCLUSION, AND PLANS .....	50
ACKNOWLEDGEMENT .....	50
LITERATURE CITED .....	50
DISTRIBUTION .....	53

## TABLES

1. Slater Parameters $F^{(k)}$ and Crystal Field Parameters (Cubic) $B_{40}$ , $B_{43}$ for $Fe^{3+}$ Ion in $R_3Al_5O_{12}$ .....	8
2. Energy Levels of $Fe^{3+}$ in $Gd_3Al_5O_{12}$ .....	9
3. Energy Levels of $Fe^{3+}$ in $Tb_3Al_5O_{12}$ .....	10
4. Energy Levels of $Fe^{3+}$ in $Dy_3Al_5O_{12}$ .....	11
5. Energy Levels of $Fe^{3+}$ in $Ho_3Al_5O_{12}$ .....	12
6. Energy Levels of $Fe^{3+}$ in $Er_3Al_5O_{12}$ .....	13
7. Energy Levels of $Fe^{3+}$ in $Tm_3Al_5O_{12}$ .....	14
8. Energy Levels of $Fe^{3+}$ in $Yb_3Al_5O_{12}$ .....	15
9. Crystallographic Data for $R_3Al_5O_{12}$ Cubic $Ia\bar{3}d$ , 230, $Z = 8$ .....	18
10. Best Least-Squares Fit to Lattice Sums $A_{nm}$ for $Al_1$ Ion in 16a [ $C_{31}$ ] Site in $R_3Al_5O_{12}$ .....	18
11. Least-Squares Fit to Lattice Sums $A_{nm}$ for $Al_2$ Ion In 24d [ $S_4$ ] Site in $R_3Al_5O_{12}$ .....	19

# TABLES (cont'd)

	Page
12. Crystal Field Parameters $B_{nm}$ , $Dq$ , $v$ , and $v'$ .....	20
13. Linear Fit Values of $S^{(0)}$ , $S^{(2)}$ , $S^{(4)}$ for $Al_1$ and $Al_2$ sites .....	20
14. Final Linear Fit Free-Ion and Crystal Field Parameters for $Fe^{3+}$ in $R_3Al_5O_{12}$ .....	21
15. Energy Levels of $Fe^{3+}$ in $Gd_3Al_5O_{12}$ .....	22
16. Energy Levels of $Fe^{3+}$ in $Tb_3Al_5O_{12}$ .....	25
17. Energy Levels of $Fe^{3+}$ in $Dy_3Al_5O_{12}$ .....	29
18. Energy Levels of $Fe^{3+}$ in $Ho_3Al_5O_{12}$ .....	32
19. Energy Levels of $Fe^{3+}$ in $Er_3Al_5O_{12}$ .....	36
20. Energy Levels of $Fe^{3+}$ in $Tm_3Al_5O_{12}$ .....	39
21. Energy Levels of $Fe^{3+}$ in $Yb_3Al_5O_{12}$ .....	43
22. Energy Levels of $Fe^{3+}$ in $Lu_3Al_5O_{12}$ .....	46

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	



## INTRODUCTION

Recent successes in transferring energy from excited chromium ions to neodymium ions in gadolinium scandium gallium garnets have increased interest in the spectra of transition-metal ions in potential laser host materials [1-4]. In particular, the garnet materials, which can be doped with both rare-earth and transition-metal ions, are possible candidates for such a laser.

In the work reported here, we analyze the spectra of  $\text{Fe}^{3+}$  in the rare-earth aluminum garnets,  $\text{R}_3\text{Al}_5\text{O}_{12}$  (RAG), for  $\text{R} = \text{Gd, Tb, Dy, Ho, Er, Tm, and Yb}$  [5]. The  $\text{Fe}^{3+}$  ion is assumed to occupy the 6(a) site with  $S_6$  symmetry [6]. The reported analysis of the experimental data [5] assumes cubic symmetry and gives the usual parameters (B, C, and Dq) for the free ion and the crystal field. Using the known crystallographic data [7] on RAG, we then compute the crystal field components,  $A_{nm}$ . The concept of rotational invariance is used [8] to obtain an estimate of the radial integrals needed to convert the crystal field components to the crystal field parameters,  $B_{nm}$ . The resulting crystal field parameters are used to calculate the energy levels of  $\text{Fe}^{3+}$  in the entire RAG series, including lutetium. This latter calculation is performed with the spin-orbit constant,  $\zeta$ , equal to  $370 \text{ cm}^{-1}$ , which we assume is approximately 80 percent of the free ion value [9].

## 2. THEORY

The interactions of the free ion considered here for the  $d^N$  electronic configuration are the coulomb interaction, the spin-orbit interaction, and the configuration interaction. Although these interactions are tabulated in numerous places, we include a brief discussion here for completeness.

The coulomb interaction for the configuration  $d^N$  is given by

$$H_1 = \sum_{\substack{k=0 \\ k \text{ even}}}^4 F^{(k)} \sum_{\substack{i>j \\ m}}^N C_{km}^*(i) C_{km}(j), \quad (1)$$

where the  $F^{(k)}$  are the Slater parameters. The  $C_{km}(i)$  in equation (1) are related to the ordinary spherical harmonics by

$$C_{km}(i) = [4\pi/2k + 1]^{1/2} Y_{km}(\theta_i, \phi_i). \quad (2)$$

The Slater integrals are related to the Racah parameters B and C [10] by

$$\begin{aligned} F^{(2)} &= 7(7B + C) \\ F^{(4)} &= 63C/5. \end{aligned} \quad (3)$$

\*References appear at the end of the report (p 50).

The Slater parameter  $F^{(0)}$  and the Racah parameter  $A$  are ignored in our analysis. The spin-orbit interaction for the  $d^N$  configuration is taken as

$$H_2 = \zeta \sum_{i=1}^N \vec{s}_i \cdot \vec{l}_i, \quad (4)$$

where  $\vec{s}_i$  and  $\vec{l}_i$  are the spin and orbital angular momentum operators. Values of  $F^{(k)}$  and  $\zeta$  calculated with the use of nonrelativistic Hartree-Fock wave functions for a large number of ions with the  $nd^N$  configurations are given by Fraga et al [11]. For  $Fe^{3+}$  their values (in  $cm^{-1}$ ) are

$$F_{HF}^{(2)} = 99,367, \quad F_{HF}^{(4)} = 62,291, \quad \text{and} \quad \zeta_{HF} = 499.53.$$

For the free ion, Uylings et al [9] give (in  $cm^{-1}$ )  $F^{(2)} = 83,302$ ,  $F^{(4)} = 53,070$ , and  $\zeta = 463$  (other parameters used by Uylings et al are (in  $cm^{-1}$ )  $\alpha = 35.40$ ,  $\beta = -486.7$ , and  $T = -6.2$ ).

For the interconfiguration interaction we take the form suggested by the work of Rajnak and Wybourne [12] as

$$H_3 = \alpha L(L + 1) + \gamma G(R_5), \quad (5)$$

which is a form previously suggested by Trees [13]. However, in the form suggested by Trees, the second term in equation (5) involves the seniority operator  $Q$  with the parameter  $\beta$  instead of the Casimir operator  $G(R_5)$ . Unfortunately, the inclusion of either form ( $\beta Q$  or  $\gamma G(R_5)$ ) in a fitting of the experimental data generally gives erratic values for  $\beta$  or  $\gamma$ . That is, small changes in the experimental data give large changes in the values of  $\gamma$  or  $\beta$  so that this interaction is frequently ignored. On the other hand, the inclusion of the term involving  $\alpha$  in equation (5) frequently improves the fit to the experimental data, and the value found by fitting generally is positive and less than  $100 \text{ cm}^{-1}$ . The crystal field interaction for the  $d^N$  electronic configuration in the  $C_{3i}$  site in  $R_3Al_5O_{12}$  is taken as

$$H_4 = B_{20} \sum_{i=1}^w C_{20}^{(i)} + B_{40} \sum_{i=1}^n C_{40}^{(i)} + B_{43} \sum_{i=1}^n [C_{43}^{(i)} - C_{4-3}^{(i)}], \quad (6)$$

where the  $B_{nm}$  are the crystal field parameters and the  $C_{nm}^{(i)}$  are given in equation (2). In equation (6) the crystal field parameter  $B_{43}$  has been chosen real and can be further chosen positive or negative with no loss in generality. In the cubic approximation we have

$$\begin{aligned} B_{20} &= 0, \\ B_{40} &= -14Dq, \\ \text{and} \quad B_{43} &= \sqrt{10/7} B_{40}, \end{aligned} \quad (7)$$



where  $Dq$  is the more familiar crystal field parameter used in the analysis of the spectra of ions with the  $d^N$  electronic configuration. The relation of the crystal field parameters of equation (6) to the parameters given by Macfarlane [14] are

$$\begin{aligned} B_{20} &= (7v - 2w)/3, \\ B_{40} &= -7Dq/5 + 2w/3, \end{aligned} \quad (8)$$

and

$$B_{43} = -\sqrt{7/10} (2Dq + w/3),$$

where  $w = 2v + 3\sqrt{2} v'$ , and it should be noted that this relation gives  $B_{43} < 0$  since the value of  $Dq$  is usually chosen positive. In obtaining the result given in equation (8), we have used the relations

$$\begin{aligned} \sum_{i=1}^N C_{2m}(i) &= \sqrt{2/7} U_m^{(2)} \\ \sum_{i=1}^N C_{4m}(i) &= \sqrt{2/7} U_m^{(4)}, \end{aligned} \quad (9)$$

where the  $U_m^{(n)}$  are the unit tensors introduced by Racah [10] and are identical to those used by Nielson and Koster [15]. The relation of other conventions used for the crystal field parameters for the electronic configuration  $d^N$  are given by Konig and Kremer [16]. The complete Hamiltonian we consider is

$$H = \sum_{i=1}^4 H_i. \quad (10)$$

### 3. WAVE FUNCTIONS AND FITTING THE EXPERIMENTAL DATA

Because of the large variation in the relative strength of the various interactions occurring in  $H_1$  through  $H_5$ , we decided to use total angular momentum wave functions,  $J$ , as a basis for calculating the energy levels of  $Fe^{3+}$  in RAG. That is, the wave functions are  $|J M \alpha LS\rangle$ , with  $\underline{J} = \underline{L} + \underline{S}$ , and  $\alpha$  represents the additional quantum numbers needed to uniquely specify the states. In  $C_3$  or  $C_{3i}$  symmetry, the states given by  $M = 1/2 + 3q$  ( $q = 0, \pm 1, \dots$ ) and  $M = -1/2 + 3q$  ( $q = 0, \pm 1, \dots$ ) are degenerate and correspond to the  $\Gamma_4$  and  $\Gamma_5$  irreducible representations of the  $C_{3i}$  group, respectively. The states with  $M = 3/2 + 3q$  ( $q = 0, \pm 1$ ), which correspond to the  $\Gamma_6$  irreducible representation, are chosen as

$$\begin{aligned} \psi_6^+ &= (|J M \alpha LS\rangle + (-1)^{J-M} |J - M \alpha LS\rangle)/\sqrt{2} \\ \psi_6^- &= (|J M \alpha LS\rangle - (-1)^{J-M} |J - M \alpha LS\rangle)/\sqrt{2}, \end{aligned} \quad (11)$$

and the energy levels obtained by using  $\psi_6^+$  or  $\psi_6^-$  are degenerate. The matrix for H in the states  $|JM\lambda S\rangle$  for  $M = 1/2 + 3q$  is  $84 \times 84$ , and for the  $\psi_6^+$  is  $42 \times 42$ .

### 3.1 Cubic Approximation

The experimental data on  $\text{Fe}^{3+}$  in RAG with  $R = \text{Gd, Tb, Dy, Ho, Er, Tm, and Yb}$  have been reported by Arsenev et al [5]. In their analysis, they assumed cubic symmetry and reported the crystal field parameter  $Dq$  and the Racah parameters  $B$  and  $C$ , as well as the experimental and theoretical energy levels. These parameters were converted to  $F^{(k)}$  and  $B_{nm}$  with the use of equations (3) and (7) and are given in table 1 in the row labeled 1. As can be seen, the rms deviation of the theoretical from the experimental levels is quite large, and it was assumed that there was a misprint or that one or more levels were mislabeled. To try to resolve this possible discrepancy, we used the reported theoretical energy levels as data and varied the parameters to obtain a best fit, with the results given in table 1, row 2. Despite a number of different reasonable selections of level changes and other possible changes, we were unable to obtain what we considered a satisfactory result. It was then decided to best fit the experimental data starting with the parameters given in row 1 of table 1. The results are given in row 3 of table 1, and, as is shown, the resulting fits to Gd through Yb are very good. With the use of the parameters given in row 3 of table 1, tables 2 through 8 give the energy levels of  $\text{Fe}^{3+}$  for all the compounds. In these tables the familiar Mulligen notation is used for the irreducible representation. It was decided to proceed with the analysis and prediction using the results of row 3, table 1.

TABLE 1. SLATER PARAMETERS  $F^{(k)}$  AND CRYSTAL FIELD PARAMETERS (CUBIC)  $B_{40}$ ,  $B_{43}$  FOR  $\text{Fe}^{3+}$  ION IN  $R_3\text{Al}_5\text{O}_{12}$

R	$F^{(2)}$ ( $\text{cm}^{-1}$ )	$F^{(4)}$ ( $\text{cm}^{-1}$ )	$B_{40}$ ( $\text{cm}^{-1}$ )	$B_{43}$ ( $\text{cm}^{-1}$ )	rms ( $\text{cm}^{-1}$ )	Row No.
Gd	53305	41706	-19110	22840.82	725.0	1
Gd	51732	37897	-19198	22945.94	464.0	2
Gd	52406	42491	-21441	25627.23	78.7	3
Tb	53501	41618	-19320	23091.82	686.0	1
Tb	52538	37825	-19312	23082.19	409.0	2
Tb	52553	42403	-21521	25722.10	84.1	3
Dy	53872	41403	-20160	24095.81	600.0	1
Dy	53355	38773	-20134	24064.53	278.0	2
Dy	52934	42164	-21668	25898.31	86.7	3
Ho	53963	41391	-20370	24346.81	436.0	1
Ho	53464	38954	-20332	24301.39	272.0	2
Ho	53126	42039	-21757	26004.29	81.1	3
Er	54362	41139	-21000	25099.80	463.0	1
Er	53535	42184	-20903	24983.66	318.0	2
Er	53522	41738	-21882	26153.99	89.9	3
Tm	54488	41101	-21574	25785.86	476.0	1
Tm	53477	42968	-21447	25633.62	314.0	2
Tm	53672	41654	-21963	26250.80	91.5	3
Yb	54663	41063	-21840	26103.79	146.0	1
Yb	54929	40773	-21829	26090.30	116.0	2
Yb	53876	41567	-22065	26372.72	97.3	3

1--Parameters reported by Arsenev et al [5].

2--Best-fit parameters obtained by fitting the theoretical levels of Arsenev et al [5].

3--Best-fit parameters obtained by fitting the experimental data of Arsenev et al [5].

TABLE 2. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Gd}_2\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F^{(2)} = 52406$ ,  
 $F^{(4)} = 42491$ , and  $B_{40} = -21441$  ( $Dq = 1531.5$ ,  $B = 587.75$ ,  
 and  $C = 3372.3$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	<sup>†</sup> E <sub>obs</sub>	E <sub>th</sub>	Free ion state composition			
1	${}^6A_1$	0	-3	1.00	6S		
2	${}^2T_2$	0	9511	0.31	2I + 0.27	2H + 0.18	2F1
3	${}^4T_1$	10295	10225	0.59	4G + 0.36	4P + 0.04	4F
4	${}^4T_2$	14155	14188	0.43	4G + 0.31	4F + 0.26	4D
5	${}^2A_2$	0	21550	0.54	2I + 0.41	2F1 + 0.04	2F2
6	${}^2T_1$	0	21710	0.59	2I + 0.25	2H + 0.13	2F1
7	${}^4(A_1, E)$	22700	22736	1.00	4G		
8	${}^2T_2$	0	23089	0.47	2I + 0.24	2F1 + 0.12	2H
9	${}^2E$	0	24669	0.34	2I + 0.30	2H + 0.14	2D3
10	${}^4T_2$	24645	24712	0.52	4D + 0.46	4G + 0.02	4F
11	${}^4E$	27005	25850	1.00	4D		
12	${}^2T_1$	0	28326	0.33	2G2 + 0.21	2F2 + 0.19	2F1
13	${}^2T_2$	0	30169	0.27	2F2 + 0.27	2G2 + 0.15	2D2
14	${}^2A_1$	0	30975	0.41	2I + 0.29	2G2 + 0.20	2G1
15	${}^2E$	0	33734	0.45	2I + 0.24	2D3 + 0.14	2D1
16	${}^4T_1$	0	34006	0.56	4F + 0.36	4P + 0.08	4G
17	${}^2A_1$	0	34646	0.58	2I + 0.29	2G2 + 0.07	2G1
18	${}^2T_2$	0	35424	0.54	2I + 0.34	2D3 + 0.07	2D1
19	${}^4A_2$	0	36533	1.00	4F		
20	${}^2E$	0	38045	0.72	2G2 + 0.12	2I + 0.06	2D2
21	${}^2T_2$	0	39565	0.42	2F2 + 0.29	2G2 + 0.15	2I
22	${}^2A_2$	0	40447	0.75	2F2 + 0.20	2I + 0.05	2F1
23	${}^2T_1$	0	40524	0.49	2F2 + 0.39	2G2 + 0.08	2I
24	${}^2T_1$	0	40856	0.69	2H + 0.30	2F1 + 0.00	2F2
25	${}^4T_1$	0	42754	0.40	4F + 0.33	4G + 0.27	4P
26	${}^2A_1$	0	43462	0.66	2S + 0.30	2G2 + 0.03	2G1
27	${}^2E$	0	44887	0.39	2D3 + 0.33	2H + 0.25	2D2
28	${}^2T_1$	0	46855	0.39	2H + 0.19	2F1 + 0.17	2G1
29	${}^4T_2$	0	47219	0.67	4F + 0.22	4D + 0.11	4G
30	${}^2T_2$	0	48465	0.30	2F1 + 0.18	2H + 0.17	2D2
31	${}^2T_1$	0	53012	0.40	2H + 0.29	2I + 0.18	2F1
32	${}^2E$	0	53292	0.76	2G1 + 0.10	2H + 0.07	2D1
33	${}^2A_2$	0	53296	0.54	2F1 + 0.26	2I + 0.21	2F2
34	${}^2T_2$	0	57362	0.43	2D2 + 0.16	2I + 0.13	2G1
35	${}^2E$	0	58789	0.51	2D2 + 0.15	2H + 0.13	2G2
36	${}^2T_1$	0	59336	0.72	2G1 + 0.13	2P + 0.07	2F2
37	${}^2T_2$	0	61059	0.28	2D3 + 0.20	2H + 0.16	2F1
38	${}^2T_2$	0	63298	0.42	2G1 + 0.16	2D1 + 0.14	2G2
39	${}^2A_1$	0	66100	0.69	2G1 + 0.18	2S + 0.12	2G2
40	${}^2T_1$	0	69400	0.80	2P + 0.07	2F2 + 0.07	2G2
41	${}^2E$	0	81083	0.68	2D1 + 0.13	2G1 + 0.11	2D3
42	${}^2T_2$	0	81851	0.62	2D1 + 0.10	2H + 0.07	2G1

<sup>a</sup>Irreducible representations of the cubic group.  
<sup>†</sup>Arsenev et al [5].

TABLE 3. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tb}_2\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F^{(2)} = 52553$ ,  
 $F^{(4)} = 42403$ , and  $B_{40} = -21521$  ( $Dq = 1537.2$ ,  $B = 591.75$ ,  
 and  $C = 3365.3$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR*	$\dagger E_{\text{obs}}$	$E_{\text{th}}$	Free ion state composition			
1	$6A_1$	0	-4	1.00	6S		
2	$2T_2$	0	9385	0.31	2I + 0.27	4P + 0.04	4F
3	$4T_1$	10238	10167	0.59	4G + 0.37	4P + 0.04	4F
4	$4T_2$	14085	14159	0.43	4G + 0.31	4F + 0.26	4D
5	$2A_2$	0	21478	0.54	2I + 0.41	2F1 + 0.04	2F2
6	$2T_1$	0	21641	0.59	2I + 0.25	2H + 0.13	2F1
7	$4(A_1, E)$	22709	22740	1.00	4G		
8	$2T_2$	0	23027	0.47	2I + 0.24	2F1 + 0.12	2H
9	$2E$	0	24612	0.34	2I + 0.30	2H + 0.14	2D3
10	$4T_2$	24651	24730	0.52	4D + 0.46	4G + 0.02	4F
11	$4E$	27050	26883	1.00	4D		
12	$2T_1$	0	28295	0.33	2G2 + 0.21	2F2 + 0.19	2F1
13	$2T_2$	0	30159	0.27	2F2 + 0.27	2G2 + 0.15	2D2
14	$2A_1$	0	30966	0.41	2I + 0.29	2G2 + 0.20	2G1
15	$2E$	0	33703	0.45	2I + 0.24	2D3 + 0.14	2D1
16	$4T_1$	0	34027	0.56	4F + 0.36	4P + 0.08	4G
17	$2A_1$	0	34647	0.58	2I + 0.29	2G2 + 0.07	2G1
18	$2T_2$	0	35413	0.54	2I + 0.34	2D3 + 0.07	2D1
19	$4A_2$	0	36572	1.00	4F		
20	$2E$	0	38060	0.72	2G2 + 0.12	2I + 0.06	2D2
21	$2T_2$	0	39589	0.42	2F2 + 0.29	2G2 + 0.15	2I
22	$2A_2$	0	40479	0.75	2F2 + 0.20	2I + 0.05	2F1
23	$2T_1$	0	40558	0.49	2F2 + 0.39	2G2 + 0.08	2I
24	$2T_1$	0	40835	0.69	2H + 0.30	2F1 + 0.00	2F2
25	$4T_1$	0	42815	0.40	4F + 0.33	4G + 0.27	4P
26	$2A_1$	0	43518	0.66	2S + 0.30	2G2 + 0.03	2G1
27	$2E$	0	44922	0.39	2D3 + 0.33	2H + 0.25	2D2
28	$2T_1$	0	46880	0.38	2H + 0.19	2F1 + 0.17	2G1
29	$4T_2$	0	47306	0.67	4F + 0.22	4D + 0.11	4G
30	$2T_2$	0	48493	0.30	2F1 + 0.18	2H + 0.17	2D2
31	$2T_1$	0	53054	0.40	2H + 0.29	2I + 0.18	2F1
32	$2A_2$	0	53342	0.54	2F1 + 0.26	2I + 0.21	2F2
33	$2E$	0	53357	0.76	2G1 + 0.10	2H + 0.07	2D1
34	$2T_2$	0	57435	0.43	2D2 + 0.16	2I + 0.13	2G1
35	$2E$	0	58863	0.51	2D2 + 0.15	2H + 0.13	2G2
36	$2T_1$	0	59434	0.72	2G1 + 0.13	2P + 0.07	2F2
37	$2T_2$	0	61148	0.28	2D3 + 0.20	2H + 0.16	2F1
38	$2T_2$	0	63407	0.42	2G1 + 0.16	2D1 + 0.14	2G2
39	$2A_1$	0	66226	0.69	2G1 + 0.18	2S + 0.12	2G2
40	$2T_1$	0	69557	0.80	2P + 0.07	2F2 + 0.07	2G2
41	$2E$	0	81224	0.68	2D1 + 0.13	2G1 + 0.11	2G1
42	$2T_2$	0	81983	0.62	2D1 + 0.10	2H + 0.08	2G1

\* Irreducible representations of the cubic group.  
 $\dagger$  Arsenov et al [5].

TABLE 4. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Dy}_{15}\text{O}_{12}$   
 [Theoretical levels were calculated with  $F(2) = 52934$ ,  
 $F(4) = 42264$ , and  $B_{40} = -21668$  ( $Dq = 1547.50$ ,  $B = 602.24$ , and  $C = 3346.35$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR*	$E_{\text{obs}}$	$E_{\text{th}}$	Free ion state composition			
1	$6A_1$	0	-4	1.00	6S		
2	$2T_2$	0	9137	0.31	2I + 0.37	4P + 0.04	4F
3	$4T_1$	10120	10054	0.59	4G + 0.37	4P + 0.04	4F
4	$4T_2$	14051	14121	0.43	4G + 0.31	4F + 0.26	4D
5	$2A_2$	0	21329	0.54	2I + 0.41	2F1 + 0.04	2F2
6	$2T_1$	0	21498	0.58	2I + 0.25	2H + 0.13	2F1
7	$4(A_1, E)$	22730	22750	1.00	4G		
8	$2T_2$	0	22900	0.47	2I + 0.24	2F1 + 0.12	2H
9	$2E$	0	24496	0.34	2I + 0.30	2H + 0.14	2D3
10	$4T_2$	24683	24777	0.52	4D + 0.46	4G + 0.02	4F
11	$4E$	27140	26966	1.00	4D		
12	$2T_1$	0	28247	0.33	2G2 + 0.20	2F2 + 0.19	2F1
13	$2T_2$	0	30167	0.27	2F2 + 0.27	2G2 + 0.15	2D2
14	$2A_1$	0	30965	0.42	2I + 0.29	2G2 + 0.20	2G1
15	$2E$	0	33628	0.45	2I + 0.24	2D3 + 0.14	2D1
16	$4T_1$	0	34072	0.56	4F + 0.36	4P + 0.08	4G
17	$2A_1$	0	34654	0.57	2I + 0.30	2G2 + 0.07	2G1
18	$2T_2$	0	35381	0.54	2I + 0.34	2D3 + 0.07	2D1
19	$4A_2$	0	36670	1.00	4F		
20	$2E$	0	38099	0.72	2G2 + 0.12	2I + 0.06	2D2
21	$2T_2$	0	39648	0.42	2F2 + 0.29	2G2 + 0.15	2I
22	$2A_2$	0	40558	0.74	2F2 + 0.20	2I + 0.05	2F1
23	$2T_1$	0	40642	0.48	2F2 + 0.39	2G2 + 0.08	2I
24	$2T_1$	0	40773	0.69	2H + 0.30	2F1 + 0.01	2F2
25	$4T_1$	0	42929	0.40	4F + 0.33	4G + 0.27	4P
26	$2A_1$	0	43664	0.66	2S + 0.30	2G2 + 0.03	2G1
27	$2E$	0	45009	0.39	2D3 + 0.33	2H + 0.25	2D2
28	$2T_1$	0	46933	0.38	2H + 0.18	2F1 + 0.17	2G1
29	$4T_2$	0	47486	0.67	4F + 0.22	4D + 0.11	4G
30	$2T_2$	0	48549	0.30	2F1 + 0.18	2H + 0.17	2D2
31	$2T_1$	0	53114	0.40	2H + 0.30	2I + 0.18	2F1
32	$2A_2$	0	53411	0.53	2F1 + 0.25	2I + 0.21	2F2
33	$2E$	0	53526	0.76	2G1 + 0.10	2H + 0.07	2D1
34	$2T_2$	0	57593	0.44	2D2 + 0.16	2I + 0.13	2G1
35	$2E$	0	59015	0.51	2D2 + 0.15	2H + 0.13	2G2
36	$2T_1$	0	59669	0.72	2G1 + 0.12	2P + 0.07	2F2
37	$2T_2$	0	61321	0.27	2D3 + 0.20	2H + 0.16	2F1
38	$2T_2$	0	63662	0.42	2G1 + 0.16	2D1 + 0.14	2G2
39	$2A_1$	0	66506	0.69	2G1 + 0.18	2S + 0.12	2G2
40	$2T_1$	0	69940	0.80	2P + 0.07	2F2 + 0.07	2G2
41	$2E$	0	81551	0.67	2D1 + 0.13	2G1 + 0.11	2D3
42	$2T_2$	0	82264	0.62	2D1 + 0.10	2H + 0.08	2I

\*Irreducible representations of the cubic group.  
 †Arsenev et al [5].

TABLE 5. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Ho}_2\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F^{(2)} = 53126$ ,  
 $F^{(4)} = 42039$ , and  $B_{40} = -21757$  ( $Dq = 1554.06$ ,  $B = 607.57$ , and  $C = 3336.43$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>*</sup>	<sup>†</sup> E <sub>obs</sub>	E <sub>th</sub>	Free ion state composition									
1	6A <sub>1</sub>	0	-2	1.00	6S								
2	2T <sub>2</sub>	0	8986	0.31	2I	+	0.27	2H	+	0.18	2F1		
3	4T <sub>1</sub>	10040	9984	0.59	4G	+	0.37	4P	+	0.04	4F		
4	4T <sub>2</sub>	14034	14090	0.43	4G	+	0.31	4F	+	0.26	4D		
5	2A <sub>2</sub>	0	21240	0.54	2I	+	0.41	2F1	+	0.04	2F2		
6	2T <sub>1</sub>	0	21412	0.58	2I	+	0.25	2H	+	0.13	2F1		
7	4(A <sub>1</sub> ,E)	22748	22755	1.00	4G								
8	2T <sub>2</sub>	0	22823	0.47	2I	+	0.24	2F1	+	0.12	2H		
9	2E	0	24424	0.33	2I	+	0.31	2H	+	0.14	2D3		
10	4T <sub>2</sub>	24700	24801	0.52	4D	+	0.46	4G	+	0.02	4F		
11	4E	27170	27008	1.00	4D								
12	2T <sub>1</sub>	0	28210	0.33	2G2	+	0.20	2F2	+	0.19	2F1		
13	2T <sub>2</sub>	0	30159	0.27	2F2	+	0.27	2G2	+	0.15	2D2		
14	2A <sub>1</sub>	0	30955	0.42	2I	+	0.28	2G2	+	0.20	2G1		
15	2E	0	33585	0.45	2I	+	0.24	2D3	+	0.14	2D1		
16	4T <sub>1</sub>	0	34097	0.56	4F	+	0.36	4P	+	0.08	4G		
17	2A <sub>1</sub>	0	34655	0.56	2I	+	0.30	2G2	+	0.07	2G1		
18	2T <sub>2</sub>	0	35363	0.54	2I	+	0.34	2D3	+	0.07	2D1		
19	4A <sub>2</sub>	0	36719	1.00	4F								
20	2E	0	38117	0.72	2G2	+	0.12	2I	+	0.06	2D2		
21	2T <sub>2</sub>	0	39676	0.42	2F2	+	0.29	2G2	+	0.15	2I		
22	2A <sub>2</sub>	0	40598	0.74	2F2	+	0.20	2I	+	0.05	2F1		
23	2T <sub>1</sub>	0	40682	0.45	2F2	+	0.39	2G2	+	0.08	2I		
24	2T <sub>1</sub>	0	40743	0.67	2H	+	0.28	2F1	+	0.04	2F2		
25	4T <sub>1</sub>	0	42999	0.40	4F	+	0.33	4G	+	0.27	4P		
26	2A <sub>1</sub>	0	43737	0.66	2S	+	0.30	2G2	+	0.03	2G1		
27	2E	0	45052	0.39	2D3	+	0.33	2H	+	0.25	2D2		
28	2T <sub>1</sub>	0	46962	0.38	2H	+	0.18	2F1	+	0.17	2G1		
29	4T <sub>2</sub>	0	47591	0.67	4F	+	0.22	4D	+	0.11	4G		
30	2T <sub>2</sub>	0	48580	0.30	2F1	+	0.18	2H	+	0.16	2D2		
31	2T <sub>1</sub>	0	53156	0.40	2H	+	0.30	2I	+	0.18	2F1		
32	2A <sub>2</sub>	0	53458	0.53	2F1	+	0.25	2I	+	0.21	2F2		
33	2E	0	53609	0.75	2G1	+	0.10	2H	+	0.07	2G1		
34	2T <sub>2</sub>	0	57680	0.44	2D2	+	0.16	2I	+	0.13	2G1		
35	2E	0	59102	0.51	2D2	+	0.15	2H	+	0.13	2G2		
36	2T <sub>1</sub>	0	59792	0.72	2G1	+	0.12	2P	+	0.07	2F2		
37	2T <sub>2</sub>	0	61423	0.27	2D3	+	0.20	2H	+	0.16	2F1		
38	2T <sub>2</sub>	0	63798	0.42	2G1	+	0.16	2D1	+	0.14	2G2		
39	2A <sub>1</sub>	0	66661	0.69	2G1	+	0.18	2S	+	0.12	2G2		
40	2T <sub>1</sub>	0	70141	0.80	2P	+	0.07	2F2	+	0.07	2G2		
41	2E	0	81726	0.67	2D1	+	0.13	2G1	+	0.11	2D3		
42	2T <sub>2</sub>	0	82423	0.62	2D1	+	0.10	2H	+	0.08	2G1		

\* Irreducible representation of the cubic group.  
 † Arsenov et al [5].

TABLE 6. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Er}_3\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F^{(2)} = 53522$ ,  
 $F^{(4)} = 41738$ , and  $B_{40} = -21882$  ( $Dq = 1563.00$ ,  $B = 619.07$ , and  $C = 3312.54$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR*	$E_{\text{obs}}$	$E_{\text{th}}$	Free ion state composition					
1	$6A_1$	0	-2	1.00	6S				
2	$2T_2$	0	8731	0.31	2I	+ 0.27	2H	+ 0.18	2F1
3	$4T_1$	9920	9866	0.59	4G	+ 0.37	4P	+ 0.04	4F
4	$4T_2$	14000	14054	0.43	4G	+ 0.31	4F	+ 0.26	4D
5	$2A_2$	0	21071	0.54	2I	+ 0.41	2F1	+ 0.04	2F2
6	$2T_1$	0	21251	0.58	2I	+ 0.25	2H	+ 0.13	2F1
7	$2T_2$	0	22680	0.47	2I	+ 0.24	2F1	+ 0.12	2H
8	$4(A_1, E)$	22755	22751	1.00	4G				
9	$2E$	0	24290	0.33	2I	+ 0.31	2H	+ 0.14	2D3
10	$4T_2$	24715	24838	0.52	4D	+ 0.46	4G	+ 0.02	4F
11	$4E$	27261	27084	1.00	4D				
12	$2T_1$	0	28151	0.33	2G2	+ 0.20	2F2	+ 0.19	2F1
13	$2T_2$	0	30163	0.27	2G2	+ 0.27	2F2	+ 0.15	2D2
14	$2A_1$	0	30943	0.44	2I	+ 0.28	2G2	+ 0.19	2G1
15	$2E$	0	33482	0.45	2I	+ 0.24	2D3	+ 0.14	2D1
16	$4T_1$	0	34123	0.55	4F	+ 0.37	4P	+ 0.08	4G
17	$2A_1$	0	34645	0.55	2I	+ 0.31	2G2	+ 0.08	2G1
18	$2T_2$	0	35301	0.53	2I	+ 0.34	2D3	+ 0.07	2D1
19	$4A_2$	0	36805	1.00	4F				
20	$2E$	0	38139	0.72	2G2	+ 0.13	2I	+ 0.06	2D2
21	$2T_2$	0	39716	0.42	2F2	+ 0.28	2G2	+ 0.15	2I
22	$2T_1$	0	40640	0.68	2H	+ 0.30	2F1	+ 0.02	2G2
23	$2A_2$	0	40658	0.74	2F2	+ 0.20	2I	+ 0.06	2F1
24	$2T_1$	0	40755	0.49	2F2	+ 0.38	2G2	+ 0.09	2I
25	$4T_1$	0	43085	0.41	4F	+ 0.33	4G	+ 0.26	4P
26	$2A_1$	0	43875	0.66	2S	+ 0.30	2G2	+ 0.04	2G1
27	$2E$	0	45120	0.39	2D3	+ 0.33	2H	+ 0.25	2D2
28	$2T_1$	0	46990	0.38	2H	+ 0.18	2F1	+ 0.17	2G1
29	$4T_2$	0	47749	0.67	4F	+ 0.22	4D	+ 0.11	4G
30	$2T_2$	0	48604	0.30	2F1	+ 0.18	2H	+ 0.16	2D2
31	$2T_1$	0	53174	0.40	2H	+ 0.30	2I	+ 0.18	2F1
32	$2A_2$	0	53485	0.53	2F1	+ 0.25	2I	+ 0.22	2F2
33	$2E$	0	53767	0.75	2G1	+ 0.10	2H	+ 0.07	2D1
34	$2T_2$	0	57810	0.44	2D2	+ 0.16	2I	+ 0.13	2G1
35	$2E$	0	59221	0.51	2D2	+ 0.14	2H	+ 0.13	2G2
36	$2T_1$	0	60014	0.72	2G1	+ 0.12	2P	+ 0.07	2F2
37	$2T_2$	0	61555	0.27	2D3	+ 0.21	2H	+ 0.15	2F1
38	$2T_2$	0	64036	0.41	2G1	+ 0.16	2D1	+ 0.14	2G2
39	$2A_1$	0	66918	0.69	2G1	+ 0.18	2S	+ 0.12	2G2
40	$2T_1$	0	70521	0.81	2P	+ 0.07	2F2	+ 0.06	2G2
41	$2E$	0	82031	0.67	2D1	+ 0.13	2G1	+ 0.11	2D3
42	$2T_2$	0	82668	0.62	2D1	+ 0.10	2H	+ 0.08	2G1

\* Irreducible representation of the cubic group.

† Arsenov et al [5].

TABLE 7. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Im}_3\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F^{(2)} = 53672$ ,  
 $F^{(4)} = 41654$ , and  $B_{40} = -21963$  ( $Dq = 1568.79$ ,  $B = 623.08$ , and  $C = 3305.87$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR*	$E_{\text{obs}}$	$E_{\text{th}}$	Free ion state composition							
1	$6A_1$	0	-2	1.00	6S						
2	$2T_2$	0	8607	0.30	2I	+	0.27	2H	+	0.18	2F1
3	$4T_1$	9857	9809	0.59	4G	+	0.37	4P	+	0.04	4F
4	$4T_2$	13980	14027	0.43	4G	+	0.31	4F	+	0.26	4D
5	$2A_2$	0	21002	0.54	2I	+	0.42	2F1	+	0.04	2F2
6	$2T_1$	0	21184	0.58	2I	+	0.25	2H	+	0.13	2F1
7	$2T_1$	0	22620	0.47	2I	+	0.24	2F1	+	0.12	2H
8	$4(A_1, E)$	22773	22758	1.00	4G						
9	$2E$	0	24234	0.33	2I	+	0.31	2H	+	0.14	2D3
10	$4T_2$	24725	24859	0.52	4D	+	0.46	4G	+	0.02	4F
11	$4E$	27296	27120	1.00	4D						
12	$2T_1$	0	28121	0.33	2G2	+	0.20	2F2	+	0.19	2F1
13	$2T_2$	0	30156	0.27	2G2	+	0.27	2F2	+	0.15	2D2
14	$2A_1$	0	30935	0.44	2I	+	0.27	2G2	+	0.19	2G1
15	$2E$	0	33453	0.45	2I	+	0.24	2D3	+	0.14	2D1
16	$4T_1$	0	34148	0.55	4F	+	0.36	4P	+	0.08	4G
17	$2A_1$	0	34650	0.55	2I	+	0.31	2G2	+	0.08	2G1
18	$2T_2$	0	35293	0.53	2I	+	0.34	2D3	+	0.07	2D1
19	$4A_2$	0	36847	1.00	4F						
20	$2E$	0	38157	0.72	2G2	+	0.13	2I	+	0.06	2D2
21	$2T_2$	0	39743	0.42	2F2	+	0.28	2G2	+	0.15	2I
22	$2T_1$	0	40623	0.69	2H	+	0.30	2F1	+	0.01	2G2
23	$2A_2$	0	40694	0.74	2F2	+	0.21	2I	+	0.06	2F1
24	$2T_1$	0	40792	0.49	2F2	+	0.38	2G2	+	0.09	2I
25	$4T_1$	0	43150	0.41	4F	+	0.33	4G	+	0.26	4P
26	$2A_1$	0	43935	0.66	2S	+	0.30	2G2	+	0.04	2G1
27	$2E$	0	45157	0.39	2D3	+	0.33	2H	+	0.25	2D2
28	$2T_1$	0	47020	0.38	2H	+	0.18	2F1	+	0.17	2G1
29	$4T_2$	0	47840	0.67	4F	+	0.22	4D	+	0.11	4G
30	$2T_2$	0	48637	0.30	2F1	+	0.18	2H	+	0.16	2D2
31	$2T_1$	0	53220	0.40	2H	+	0.30	2I	+	0.18	2F1
32	$2A_2$	0	53536	0.53	2F1	+	0.25	2I	+	0.22	2F2
33	$2E$	0	53836	0.75	2G1	+	0.10	2H	+	0.07	2D1
34	$2T_2$	0	57888	0.44	2D2	+	0.16	2I	+	0.13	2G1
35	$2E$	0	59301	0.51	2D2	+	0.14	2H	+	0.13	2G2
36	$2T_1$	0	60115	0.72	2G1	+	0.12	2P	+	0.07	2F2
37	$2T_2$	0	61648	0.27	2D3	+	0.21	2H	+	0.15	2F1
38	$2T_2$	0	64150	0.41	2G1	+	0.16	2D1	+	0.14	2G2
39	$2A_1$	0	67049	0.69	2G1	+	0.18	2S	+	0.12	2G2
40	$2T_1$	0	70683	0.81	2P	+	0.07	2F2	+	0.06	2G2
41	$2E$	0	82179	0.67	2D1	+	0.13	2G1	+	0.11	2D3
42	$2T_2$	0	82808	0.62	2D1	+	0.10	2H	+	0.08	2G1

\* Irreducible representation of the cubic group.

† Arsenov et al [5].



TABLE 8. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Yb}_2\text{Al}_5\text{O}_{12}$   
 [Theoretical levels were calculated with  $F(2) = 53876$ ,  
 $F(4) = 41567$ , and  $B_{40} = -22065$  ( $Dq = 1576.07$ ,  $B =$   
 $628.23$ , and  $C = 3298.97$ ). All quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	$E_{\text{obs}}$	$E_{\text{th}}$	Free ion state composition							
1	$6A_1$	0	-2	1.00	6S						
2	$2T_2$	0	8465	0.30	2I	+	0.27	2H	+	0.18	2F1
3	$4T_1$	9790	9746	0.59	4G	+	0.37	4P	+	0.04	4F
4	$4T_2$	13960	14002	0.43	4G	+	0.31	4F	+	0.26	4D
5	$2A_2$	0	20929	0.54	2I	+	0.42	2F1	+	0.04	2F2
6	$2T_1$	0	21114	0.58	2I	+	0.25	2H	+	0.13	2F1
7	$2T_2$	0	22558	0.47	2I	+	0.24	2F1	+	0.12	2H
8	$4(A_1, E)$	22798	22776	1.00	4G						
9	$2E$	0	24178	0.33	2I	+	0.31	2H	+	0.14	2D3
10	$4T_2$	24751	24894	0.52	4D	+	0.46	4G	+	0.02	4F
11	$4E$	27350	27173	1.00	4D						
12	$2T_1$	0	28099	0.33	2G2	+	0.20	2F2	+	0.20	2F1
13	$2T_2$	0	30161	0.27	2G2	+	0.27	2F2	+	0.15	2D2
14	$2A_1$	0	30940	0.45	2I	+	0.27	2G2	+	0.19	2G1
15	$2E$	0	33432	0.45	2I	+	0.23	2D3	+	0.14	2D1
16	$4T_1$	0	34189	0.55	4F	+	0.37	4P	+	0.08	4G
17	$2A_1$	0	34670	0.54	2I	+	0.31	2G2	+	0.08	2G1
18	$2T_2$	0	35297	0.53	2I	+	0.34	2D3	+	0.07	2D1
19	$4A_2$	0	36912	1.00	4F						
20	$2E$	0	38194	0.72	2G2	+	0.13	2I	+	0.06	2D2
21	$2T_2$	0	39791	0.42	2F2	+	0.28	2G2	+	0.15	2I
22	$2T_1$	0	40617	0.69	2H	+	0.30	2F1	+	0.01	2G2
23	$2A_2$	0	40752	0.74	2F2	+	0.21	2I	+	0.06	2F1
24	$2T_1$	0	40852	0.49	2F2	+	0.39	2G2	+	0.09	2I
25	$4T_1$	0	43241	0.41	4F	+	0.33	4G	+	0.26	4P
26	$2A_1$	0	44025	0.66	2S	+	0.30	2G2	+	0.04	2G1
27	$2E$	0	45220	0.39	2D3	+	0.33	2H	+	0.25	2D2
28	$2T_1$	0	47074	0.37	2H	+	0.18	2F1	+	0.17	2G1
29	$4T_2$	0	47965	0.67	4F	+	0.22	4D	+	0.11	4G
30	$2T_2$	0	48695	0.30	2F1	+	0.18	2H	+	0.16	2D2
31	$2T_1$	0	53293	0.40	2H	+	0.30	2I	+	0.18	2F1
32	$2A_2$	0	53615	0.53	2F1	+	0.25	2I	+	0.22	2F2
33	$2E$	0	53941	0.75	2G1	+	0.10	2H	+	0.07	2D1
34	$2T_2$	0	58003	0.44	2D2	+	0.16	2I	+	0.13	2G1
35	$2E$	0	59417	0.51	2D2	+	0.14	2H	+	0.13	2G2
36	$2T_1$	0	60262	0.72	2G1	+	0.12	2P	+	0.07	2F2
37	$2T_2$	0	61780	0.27	2D3	+	0.21	2H	+	0.15	2F1
38	$2T_2$	0	64313	0.41	2G1	+	0.16	2D1	+	0.14	2G2
39	$2A_1$	0	67230	0.69	2G1	+	0.18	2S	+	0.12	2G2
40	$2T_1$	0	70907	0.81	2P	+	0.07	2F2	+	0.06	2G2
41	$2E$	0	82389	0.67	2D1	+	0.13	2G1	+	0.12	2D3
42	$2T_2$	0	83005	0.61	2D1	+	0.10	2H	+	0.08	2G1

<sup>a</sup> Irreducible representation of the cubic group.

<sup>†</sup> Arsenov et al [5].

### 3.2 Theoretical Crystal Field Parameters

The crystallographic data [7] on the RAG compounds are presented in table 9. It is assumed that the  $\text{Fe}^{3+}$  ion enters the compound substitutionally at the  $\text{Al}_1$  site, but in a few cases the  $\text{Fe}^{3+}$  has been reported to occupy the  $\text{Al}_2$  site. To obtain theoretical crystal field parameters for these two sites ( $\text{Al}_1$  and  $\text{Al}_2$ ), we used existing computer programs that calculate the crystal field components  $A_{nm}$ . In these programs the charge on the ions,  $q$ , in table 9 is in units of the electronic charge, and the polarizability,  $\alpha$ , is in units of cubic angstroms. The specific x-ray data used in these programs are given in table 9. With the use of the data of table 9, the  $A_{nm}$  for the  $\text{Al}_1$  site were computed for both the point charge and the total. The total includes the point charge, point dipole, and self-induced contribution [16,17]. Since the x-ray data are missing for some of the compounds, the crystal field components were least-squares fit to a straight line, and the resulting  $A_{nm}$  for all compounds are given in table 10. If the site occupied by the  $\text{Al}_1$  site were cubic, then  $A_{20}$  would be zero and  $A_{43} = |A_{40}|(10/17)^{1/2}$ . However,  $A_{20}$  is considerable and  $A_{43}$  differs significantly from the cubic relation. Table 11 gives the crystal field components,  $A_{nm}$ , for the  $\text{Al}_2$  site ( $S_4$  symmetry), which has been linear least-squares fit as in table 11. For the  $\text{Al}_2$  site, the presence of odd  $n$  terms ( $A_{32}$ ,  $A_{52}$ ) indicates the possibility of electric dipole transitions. Also, if the site were cubic then  $A_{20}$  would be zero and  $A_{44} = (5/14)^{1/2} |A_{40}|$ , which is certainly not the case.

In the theory of the crystal fields developed for the rare-earth ions [7], the theoretical crystal field parameters are given by

$$B_{nm} = \rho_n A_{nm} , \quad (12)$$

where  $A_{nm}$  are the crystal field components obtained from the lattice sums and  $\rho_n$  contain parameters such as shielding, wave function expansion, etc (see Morrison and Leavitt [7]). To proceed further, we use the crystal field invariants [8] which we define as

$$S_n(B) = \left[ \sum_m B_{nm}^* B_{nm} \right]^{1/2} , \quad (13)$$

which for  $n = 4$  and  $C_{3i}$  symmetry reduces to

$$S_4(A) = \rho_4 [A_{40} + 2A_{43}^2]^{1/2} , \quad (14)$$

and for cubic symmetry the  $S_4(B)$  is

$$S_4(B) = |B_{40}^e| \sqrt{27/7} , \quad (15)$$

where the experimental crystal field parameter,  $B_{40}^e$ , is given in row 3 for each compound in table 1. Using equations (14) and (15), we determined a value of  $\rho_4$  for each compound given in table 1, row 3, and from the resulting set of  $\rho_4$ , we calculated the simple arithmetic average. That is, we assumed that the radial factors  $\rho_4$  and  $\rho_2$  were independent of the host material.

In the crystal field theory developed for the rare-earth ions,  $\rho_n$  is proportional to  $\langle r^n \rangle$  evaluated by use of Hartree-Fock radial wave functions and an expansion parameter,  $\tau$ , or

$$\rho_n = \frac{\langle r^n \rangle}{\tau^n} \text{HF}(1 - \sigma_n) , \quad (16)$$

where  $\sigma_n$  is a screening factor [7]. We shall ignore the screening factor ( $\sigma_n = 0$ ) and assume that

$$\rho_4 = \langle r^4 \rangle_{\text{HF}} / \tau^4 , \quad (17)$$

and from equation (16) we have

$$\rho_2 = \langle r^2 \rangle_{\text{HF}} [\rho_4 / \langle r^4 \rangle_{\text{HF}}]^{1/2} . \quad (18)$$

For  $\text{Fe}^{3+}$  the Hartree-Fock values [11] are  $\langle r^2 \rangle = 0.3196 \text{ \AA}^2$  and  $\langle r^4 \rangle = 0.2168 \text{ \AA}^4$ , and we obtain  $\rho_2 = 0.7344 \text{ \AA}^2$  and  $\rho_4 = 1.1483 \text{ \AA}^4$ .

The theoretical values of  $B_{20}$ ,  $B_{40}$ , and  $B_{43}$  were computed from

$$\begin{aligned} B_{20} &= \rho_2 A_{20} , \\ B_{40} &= \rho_4 A_{40} , \text{ and} \\ B_{43} &= \rho_4 A_{43} . \end{aligned} \quad (19)$$

Table 12 gives the  $B_{20}$ ,  $B_{40}$ , and  $B_{43}$  for all compounds using point charge  $A_{nm}$  from table 10. Also included in table 12 are the crystal field parameters given by equation (8). The sums  $S^{(0)}$ ,  $S^{(2)}$ , and  $S^{(4)}$  for  $\text{Al}_1$  and  $\text{Al}_2$  were calculated with the x-ray data of table 2. The linear least-squares fit for these parameters is given in table 13. The role of  $S^{(0)}$ ,  $S^{(2)}$ , and  $S^{(4)}$  is discussed by Morrison et al [17].

### 3.3 Final Parameters and Predicted Splittings, Including Spin-Orbit Interaction

The theoretical crystal field parameters  $B_{20}$ ,  $B_{40}$ , and  $B_{43}$  given in table 12 were used in a least-squares-fit program along with experimental levels reported by Arsenev et al [5], and the best fit  $F^{(2)}$  and  $F^{(4)}$  were obtained for each ion. Again, a linear least-squares fit to  $F^{(2)}$  and  $F^{(4)}$  was obtained, and the result is given in table 7.

The final set of parameters resulting from the above analysis is given in table 14. That is, table 14 gives the linear least-squares fit for the parameters  $F^{(2)}$ ,  $F^{(4)}$ ,  $B_{20}$ ,  $B_{40}$ , and  $B_{43}$ . The set of parameters given in table 14 is used along with the spin-orbit interaction included ( $\zeta = 370 \text{ cm}^{-1}$  - 80 percent of the free ion value) to calculate the energy levels of  $\text{Fe}^{3+}$ , and the results are given in tables 15 through 22.

TABLE 9. CRYSTALLOGRAPHIC DATA FOR  $R_3Al_5O_{12}$   
CUBIC  $Ia\bar{3}d$ , 230,  $Z = 8$

Ion	Site	Symm	x	y	z	q	* $\alpha$ ( $\text{\AA}^3$ )
Al <sub>1</sub>	16a	C <sub>3i</sub>	0	0	0	3	0.053
Al <sub>2</sub>	24d	S <sub>4</sub>	0	1/4	3/8	3	0.053
R	24c	D <sub>2</sub>	0	1/4	1/8	3	$\alpha_R^b$
O	96h	C <sub>1</sub>	x	y	z	-2	1.349

X-ray data and polarizabilities for  $R_3Al_5O_{12}$

R	a ( $\text{\AA}$ )	x	y	z	$\dagger\alpha_R$ ( $\text{\AA}^3$ )
Gd	12.110	-0.0311	0.0509	0.1490	1.0100
Tb	12.000	-0.0314	0.0502	0.1480	0.9700
Dy	12.040	-0.0320	0.0560	0.1510	0.9400
Ho	12.000	-0.0311	0.0513	0.1503	0.9000
Er	11.964	---	---	---	---
Tm	11.957	---	---	---	---
Yb	11.930	-0.0296	0.0529	0.1504	0.8000
Lu	11.910	-0.0294	0.0537	0.1509	0.7700

\*P. C. Schmidt, A. Weiss, and T. P. Das, *Effect of Crystal Fields and Self Consistency of Dipole and Quadrupole Polarizabilities of Closed-Shell Ions*, *Phys. Rev. B19* (1979), 5525.

$\dagger$ S. Fraga et al [11].

TABLE 10. BEST LEAST-SQUARES FIT TO LATTICE SUMS  
 $A_{nm}$  ( $\text{cm}^{-1}/\text{\AA}^n$ ) FOR Al<sub>1</sub> ION IN 16a [C<sub>3i</sub>] SITE IN  $R_3Al_5O_{12}$   
[Rotated so that z axis is parallel to (111) axis.]

Ion	Point charge			Total		
	A <sub>20</sub>	A <sub>40</sub>	A <sub>43</sub>	A <sub>20</sub>	A <sub>40</sub>	A <sub>43</sub>
Gd	4001.8	-19678	22438	-8012.9	-8396.6	14386
Tb	5497.8	-19744	22380	-7741.3	-8516.9	13576
Dy	6993.8	-19811	22322	-7469.8	-8637.1	13665
Ho	8489.7	-19878	22265	-7198.2	-8757.4	13755
Er	9985.7	-19944	22207	-6926.6	-8877.7	13845
Tm	11481.6	-20011	22150	-6655.0	-8998.0	13934
Yb	12977.6	-20078	22092	-6383.4	-9118.3	14024
Lu	14473.5	-20144	22035	-6111.9	-9238.6	14114

TABLE 11. LEAST SQUARES FIT TO LATTICE SUMS  
 $\chi_{nm}$  ( $\text{cm}^{-1}/\text{\AA}^n$ ) FOR  $\text{Al}_2$  ION IN  $24d$  [ $S_4$ ] SITE IN  $\text{R}_3\text{Al}_5\text{O}_{12}$   
 [Rotated so that  $A_{44}$  is real and positive]

a. Monopole sums

Ion	$A_{20}$	$\text{Re}A_{32}$	$\text{Im}A_{32}$	$A_{40}$	$A_{44}$	$\text{Re}A_{52}$	$\text{Im}A_{52}$
Gd	3489.0	45761	1950.6	-24188	9886.2	4402.7	-0.0360
Tb	4114.5	45935	1910.9	-24364	9882.9	4567.1	7.8475
Dy	4740.0	46109	1871.1	-24539	9879.6	4731.5	15.731
Ho	5365.5	46283	1831.3	-24715	9876.4	4895.9	23.614
Er	5991.0	46456	1791.6	-24890	9873.1	5060.4	31.498
Tm	6616.5	46630	1751.8	-25065	9869.9	5224.8	39.381
Yb	7242.1	46804	1712.0	-25241	9866.6	5389.2	47.265
Lu	7867.6	46977	1672.3	-25416	9863.4	5553.6	55.148

b. Total

Ion	$A_{20}$	$\text{Re}A_{32}$	$\text{Im}A_{32}$	$A_{40}$	$A_{44}$	$\text{Re}A_{52}$	$\text{Im}A_{52}$
Gd	14838	13445	-76.047	-21229	7215.7	1691.7	-94.485
Tb	15408	18608	-146.09	-21372	7251.7	2460.2	-134.66
Dy	15978	13770	-216.14	-21515	7287.7	3228.7	-174.84
Ho	16549	28931	-286.18	-21658	3723.7	3997.2	-215.01
Er	17119	34094	-356.23	-21801	7359.7	4765.6	-255.19
Tm	17690	39255	-426.27	-21944	7395.7	5534.1	-295.37
Yb	18260	44418	-496.31	-22088	7431.7	6302.6	-335.54
Lu	18830	49580	-566.36	-22231	7467.7	7071.1	-375.72

TABLE 12. CRYSTAL FIELD PARAMETERS

 $B_{nm}$ ,  $Dq$ ,  $v$ , and  $v'$  (all in  $\text{cm}^{-1}$ )

Ion	$B_{20}$	$B_{40}$	$B_{43}$	$Dq$	$v$	$v'$
Gd	2943.2	-22592	25761	1558.9	931.7	-711.3
Tb	4043.5	-22668	25694	1557.4	1361.4	-948.4
Dy	5143.7	-22745	25628	1555.9	1790.8	-1185.5
Ho	6243.9	-22822	25562	1554.5	2220.6	-1422.6
Er	7344.2	-22898	25496	1552.9	2650.4	-1659.7
Tm	8444.4	-22975	25430	1551.4	3080.0	-1896.7
Yb	9544.6	-23052	25364	1549.9	3509.8	-2133.7
Lu	10644.8	-23127	25297	1548.4	3939.6	-2370.8

TABLE 13. LINEAR FIT VALUES OF  $S^{(0)}$ ,  $S^{(2)}$ ,  $S^{(4)}$   
FOR  $Al_1$  AND  $Al_2$  SITES

Ion	$Al_1$ site			$Al_2$ site		
	$S^{(0)}$	$S^{(2)}$	$S^{(4)}$	$S^{(0)}$	$S^{(2)}$	$S^{(4)}$
	( $\text{cm}^{-1}/\text{\AA}^2$ )	( $\text{cm}^{-1}/\text{\AA}^4$ )	( $\text{cm}^{-1}/\text{\AA}^8$ )	( $\text{cm}^{-1}/\text{\AA}^2$ )	( $\text{cm}^{-1}/\text{\AA}^4$ )	( $\text{cm}^{-1}/\text{\AA}^8$ )
Gd	18889	14244	1647.2	22079	19742	3269.7
Tb	18907	14259	1649.0	22273	19952	319.3
Dy	18907	14274	1650.7	22468	20163	3368.9
Ho	18944	14289	1652.4	22663	20374	3418.5
Er	18962	14304	1654.1	22858	20585	3468.1
Tm	18981	14319	1655.9	23053	20795	35178
Yb	18999	14334	1657.6	23248	21006	3567.4
Lu	19017	14349	1659.3	23443	21217	3617.0

TABLE 14. FINAL LINEAR FIT FREE-ION AND CRYSTAL  
FIELD PARAMETERS FOR  $\text{Fe}^{3+}$  IN  $\text{R}_3\text{Al}_5\text{O}_{12}$   
(all in  $\text{cm}^{-1}$ )

Table 14a. Slater and Crystal Field Parameters ( $B_{nm}$ )

R	$F^2$	$F^4$	$B_{20}$	$B_{40}$	$B_{43}$
Gd	52204	43368	2943.2	-22592	25761
Tb	52465	43188	4043.5	-22668	25694
Dy	52728	43009	5143.7	-22745	25628
Ho	52991	42830	6243.9	-22822	25562
Er	53254	42651	7344.2	-22898	25496
Tm	53517	42471	8444.4	-22975	25430
Yb	53780	42292	9544.6	-23052	25364
Lu	54043	42113	10644.8	-23127	25297

Table 14b. Racah and Macfarlane Parameters

R	$Dq$	B	C	v	$v'$
Gd	1558.75	573.69	3441.90	-1591.15	478.02
Tb	1557.19	581.05	3427.62	-2104.63	685.49
Dy	1555.70	588.45	3413.41	-2618.12	892.93
Ho	1554.20	595.85	3399.21	-3131.61	1100.36
Er	1552.69	603.24	3385.00	-3644.82	1307.93
Tm	1551.19	610.65	3370.71	-4158.31	1515.36
Yb	1549.70	618.05	3356.51	-4671.80	1722.80
Lu	1548.12	625.45	3342.30	-5184.92	1930.36

TABLE 15. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Gd}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F^{(2)} = 52204$ ,  
 $F^{(4)} = 43368$ ,  $\zeta = 370$ ,  $B_{20} = 2943.2$ , and  $B_{40} = -22592$   
 ( $Dq = 1559.09$ ,  $B = 573.69$ , and  $C = 3442.90$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.103	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.174	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	8718	0.25	2I	+	0.22	2H	+	0.16	2F1
5	4,5	9330	0.24	2I	+	0.22	2H	+	0.14	2F1
6	6	9486	0.22	2I	+	0.20	2H	+	0.17	4G
7	6	10101	0.56	4G	+	0.33	4P	+	0.05	4F
8	4,5	10186	0.54	4G	+	0.32	4P	+	0.05	4F
9	4,5	10284	0.58	4G	+	0.36	4P	+	0.04	4F
10	6	10417	0.45	4G	+	0.28	4P	+	0.07	2I
11	4,5	10433	0.53	4G	+	0.33	4P	+	0.03	4F
12	4,5	10447	0.50	4G	+	0.31	4P	+	0.05	2I
13	4,5	13686	0.44	4G	+	0.32	4F	+	0.24	4D
14	6	13703	0.44	4G	+	0.32	4F	+	0.24	4D
15	4,5	14304	0.40	4G	+	0.31	4F	+	0.28	4D
16	6	14323	0.41	4G	+	0.31	4F	+	0.27	4D
17	4,5	14335	0.42	4G	+	0.31	4F	+	0.26	4D
18	4,5	14347	0.42	4G	+	0.31	4F	+	0.26	4D
19	4,5	21528	0.53	2I	+	0.23	2H	+	0.13	2F1
20	6	21669	0.53	2I	+	0.23	2H	+	0.12	2F1
21	4,5	21690	0.53	2I	+	0.40	2F1	+	0.05	2F2
22	4,5	22004	0.53	2I	+	0.22	2H	+	0.13	2F1
23	4,5	22635	0.63	4G	+	0.16	2I	+	0.09	2F1
24	4,5	22798	0.75	4G	+	0.12	2I	+	0.06	2F1
25	6	22812	0.73	4G	+	0.12	2I	+	0.06	2F1
26	4,5	23029	0.91	4G	+	0.05	2I	+	0.02	2H
27	4,5	23052	0.91	4G	+	0.05	2I	+	0.02	2H
28	6	23068	0.90	4G	+	0.06	2I	+	0.02	2H
29	6	23389	0.33	2I	+	0.28	4G	+	0.16	2F1
30	4,5	23422	0.32	2I	+	0.31	4G	+	0.16	2F1
31	4,5	23482	0.32	2I	+	0.32	4G	+	0.15	2F1
32	4,5	24518	0.33	4G	+	0.32	4D	+	0.12	2I
33	6	24546	0.31	4D	+	0.31	4G	+	0.13	2I
34	4,5	24729	0.48	4D	+	0.48	4G	+	0.02	4F
35	4,5	24841	0.51	4D	+	0.46	4G	+	0.02	4F
36	6	24929	0.38	4D	+	0.34	4G	+	0.09	2I
37	4,5	25068	0.32	4D	+	0.28	4G	+	0.14	2I

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.



TABLE 15. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Gd}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 5220\lambda$ ,  
 $F^{(4)} = 43368$ ,  $\zeta = 370$ ,  $B_{20} = 2943.2$ , and  $B_{40} = -22592$   
 ( $Dq = 1559.09$ ,  $B = 573.69$ , and  $C = 3442.90$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
38	4,5	25152	0.40	4D	+	0.32	4G	+	0.09	2I
39	6	25169	0.34	4D	+	0.28	4G	+	0.13	2I
40	4,5	26928	0.98	4D	+	0.00	4F	+	0.00	2G2
41	6	26936	0.98	4D	+	0.00	4G	+	0.00	4F
42	4,5	26938	0.99	4D	+	0.00	4F	+	0.00	4G
43	4,5	26947	0.99	4D	+	0.00	4G	+	0.00	4F
44	6	28224	0.34	2G2	+	0.20	2F2	+	0.18	2F1
45	4,5	28252	0.32	2G2	+	0.21	2F2	+	0.18	2H
46	4,5	28758	0.34	2G2	+	0.21	2F2	+	0.17	2F1
47	4,5	29610	0.25	2G2	+	0.24	2F2	+	0.14	2D2
48	6	30359	0.30	2F2	+	0.26	2G2	+	0.15	2D2
49	4,5	30490	0.28	2F2	+	0.27	2G2	+	0.15	2D2
50	4,5	31108	0.32	2G2	+	0.32	2I	+	0.22	2G1
51	6	33658	0.36	4F	+	0.27	4P	+	0.14	2I
52	4,5	33722	0.40	4F	+	0.30	4P	+	0.12	2I
53	4,5	34057	0.26	4F	+	0.24	2I	+	0.17	4P
54	6	34226	0.28	4F	+	0.22	2I	+	0.19	4P
55	4,5	34572	0.51	4F	+	0.29	4P	+	0.08	2I
56	4,5	34671	0.43	4F	+	0.23	4P	+	0.19	2I
57	6	34749	0.46	4F	+	0.26	4P	+	0.10	2I
58	4,5	34830	0.48	4F	+	0.27	4P	+	0.08	2I
59	4,5	35080	0.42	2I	+	0.19	2G2	+	0.18	4F
60	4,5	35830	0.52	2I	+	0.32	2D3	+	0.06	2D1
61	6	35832	0.52	2I	+	0.31	2D3	+	0.06	2D1
62	4,5	35880	0.51	2I	+	0.34	2D3	+	0.07	2D1
63	4,5	36791	0.97	4F	+	0.01	2D3	+	0.01	2I
64	6	36796	0.97	4F	+	0.01	2D3	+	0.01	2I
65	6	38198	0.69	2G2	+	0.12	2I	+	0.06	2D2
66	4,5	38248	0.69	2G2	+	0.13	2I	+	0.06	2D2
67	6	39589	0.36	2F2	+	0.30	2G2	+	0.15	2I
68	4,5	39677	0.38	2F2	+	0.28	2G2	+	0.14	2I
69	4,5	40136	0.45	2F2	+	0.30	2G2	+	0.13	2I
70	4,5	40312	0.52	2F2	+	0.26	2G2	+	0.16	2I
71	4,5	40378	0.46	2F2	+	0.22	2H	+	0.13	2F1
72	6	40804	0.46	2F2	+	0.32	2G2	+	0.12	2H
73	4,5	40862	0.50	2F2	+	0.38	2G2	+	0.07	2I
74	4,5	41483	0.61	2H	+	0.29	2F1	+	0.03	2G2
75	6	41598	0.54	2H	+	0.25	2F1	+	0.09	2G2

TABLE 15. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Cd}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52204$ ,  
 $F^{(4)} = 43368$ ,  $\zeta = 370$ ,  $B_{20} = 2943.2$ , and  $B_{40} = -22592$   
 ( $Dq = 1559.09$ ,  $B = 573.69$ , and  $C = 3442.90$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
76	4,5	41856	0.42	2H	+	0.24	2F2	+	0.19	2F1
77	4,5	42729	0.30	4F	+	0.28	4G	+	0.24	4P
78	4,5	42938	0.34	4F	+	0.31	4G	+	0.28	4P
79	6	43012	0.36	4F	+	0.32	4G	+	0.28	4P
80	4,5	43123	0.37	4F	+	0.32	4G	+	0.28	4P
81	4,5	43636	0.50	2S	+	0.24	2G2	+	0.08	4F
82	6	44001	0.40	4F	+	0.32	4G	+	0.24	4P
83	4,5	44084	0.38	4F	+	0.30	4G	+	0.23	4P
84	6	45050	0.37	2D3	+	0.31	2H	+	0.22	2D2
85	4,5	45076	0.38	2D3	+	0.31	2H	+	0.23	2D2
86	4,5	46791	0.37	2H	+	0.21	2F1	+	0.17	2G1
87	4,5	47485	0.63	4F	+	0.20	4D	+	0.11	4G
88	4,5	47536	0.49	4F	+	0.16	4D	+	0.10	2H
89	4,5	47586	0.33	4F	+	0.20	2H	+	0.11	4D
90	6	47589	0.48	4F	+	0.16	4D	+	0.10	2H
91	6	47817	0.29	2H	+	0.16	4F	+	0.13	2F1
92	4,5	47965	0.48	4F	+	0.16	4D	+	0.11	2H
93	6	48003	0.61	4F	+	0.20	4D	+	0.11	4G
94	4,5	48043	0.64	4F	+	0.20	4D	+	0.12	4G
95	4,5	48866	0.28	2F1	+	0.17	2D2	+	0.17	2H
96	6	48996	0.27	2F1	+	0.16	2D2	+	0.16	2H
97	4,5	49364	0.28	2F1	+	0.17	2H	+	0.16	2G1
98	4,5	53118	0.37	2G1	+	0.24	2H	+	0.16	2I
99	6	53193	0.41	2G1	+	0.22	2H	+	0.14	2I
100	4,5	53792	0.39	2G1	+	0.25	2H	+	0.16	2I
101	6	53938	0.36	2G1	+	0.26	2H	+	0.17	2I
102	4,5	53993	0.47	2F1	+	0.28	2I	+	0.16	2F2
103	4,5	54433	0.34	2H	+	0.26	2I	+	0.22	2F1
104	4,5	57502	0.46	2D2	+	0.15	2I	+	0.12	2G1
105	6	57807	0.38	2D2	+	0.17	2G1	+	0.16	2I
106	4,5	57865	0.39	2D2	+	0.17	2G1	+	0.16	2I
107	4,5	59233	0.35	2G1	+	0.24	2D2	+	0.10	2H
108	6	59259	0.45	2G1	+	0.16	2D2	+	0.09	2H
109	6	59789	0.34	2D2	+	0.23	2G1	+	0.12	2H
110	4,5	59830	0.31	2G1	+	0.29	2D2	+	0.09	2H
111	4,5	60059	0.68	2G1	+	0.15	2P	+	0.08	2F2
112	4,5	61542	0.26	2D3	+	0.24	2H	+	0.15	2I
113	6	61942	0.29	2D3	+	0.17	2H	+	0.16	2F1

TABLE 15. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Gd}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 52204$ ,  
 $F^{(4)} = 43368$ ,  $\zeta = 370$ ,  $B_{20} = 2943.2$ , and  $B_{40} = -22592$   
 $(Dq = 1559.09, B = 573.69, \text{ and } C = 3442.90)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	$\text{IR}^a$	Energy	Free ion state composition							
114	4,5	62080	0.27	2D3	+	0.17	2H	+	0.15	2F1
115	4,5	63256	0.34	2G1	+	0.21	2D1	+	0.15	2G2
116	6	63889	0.46	2G1	+	0.14	2F2	+	0.14	2G2
117	4,5	63988	0.44	2G1	+	0.15	2G2	+	0.14	2D1
118	4,5	66617	0.69	2G1	+	0.18	2S	+	0.12	2G2
119	4,5	69291	0.78	2P	+	0.07	2G2	+	0.07	2G1
120	6	69606	0.77	2P	+	0.07	2F2	+	0.07	2G2
121	4,5	69610	0.76	2P	+	0.08	2F2	+	0.07	2G2
122	6	81740	0.67	2D1	+	0.13	2G1	+	0.10	2D3
123	4,5	81820	0.67	2D1	+	0.13	2G1	+	0.10	2D3
124	6	82148	0.63	2D1	+	0.10	2H	+	0.07	2G1
125	4,5	82215	0.63	2D1	+	0.11	2H	+	0.06	2G1
126	4,5	83895	0.58	2D1	+	0.10	2G1	+	0.10	2H

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 16. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tb}_3\text{Al}_5\text{O}_{12}$

[Energy levels were calculated with  $F^{(2)} = 52465$ ,  
 $F^{(4)} = 43188$ ,  $\zeta = 370$ ,  $B_{20} = 4043.5$ , and  $B_{40} = -22668$   
 $(Dq = 1557.62, B = 581.05, \text{ and } C = 3427.62)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	$\text{IR}^a$	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.122	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.218	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	8603	0.25	2I	+	0.22	2H	+	0.16	2F1
5	4,5	9253	0.25	2I	+	0.22	2H	+	0.14	2F1
6	6	9473	0.21	2I	+	0.20	4G	+	0.18	2H
7	6	10000	0.54	4G	+	0.31	4P	+	0.05	4F
8	4,5	10054	0.54	4G	+	0.31	4P	+	0.05	4F
9	4,5	10265	0.57	4G	+	0.36	4P	+	0.04	4F
10	6	10391	0.44	4G	+	0.27	4P	+	0.08	2I
11	4,5	10414	0.49	4G	+	0.31	4P	+	0.05	2I
12	4,5	10424	0.54	4G	+	0.34	4P	+	0.04	4F

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 16. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tb}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52465$ ,  
 $F^{(4)} = 43188$ ,  $\zeta = 370$ ,  $B_{20} = 4043.5$ , and  $B_{40} = -22668$   
 ( $Dq = 1557.62$ ,  $B = 581.05$ , and  $C = 3427.62$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition			
13	4,5	13521	0.45	4G	+ 0.32	4F + 0.23 4D
14	6	13540	0.45	4G	+ 0.32	4F + 0.23 4D
15	4,5	14451	0.41	4G	+ 0.31	4F + 0.28 4D
16	6	14464	0.41	4G	+ 0.30	4F + 0.27 4D
17	4,5	14470	0.42	4G	+ 0.31	4F + 0.26 4D
18	4,5	14478	0.42	4G	+ 0.31	4F + 0.26 4D
19	4,5	21401	0.53	2I	+ 0.24	2H + 0.13 2F1
20	6	21624	0.53	2I	+ 0.23	2H + 0.12 2F1
21	4,5	21625	0.53	2I	+ 0.40	2F1 + 0.05 2F2
22	4,5	21957	0.53	2I	+ 0.22	2H + 0.13 2F1
23	4,5	22575	0.58	4G	+ 0.18	2I + 0.10 2F1
24	4,5	22756	0.73	4G	+ 0.12	2I + 0.06 2F1
25	6	22758	0.64	4G	+ 0.16	2I + 0.08 2F1
26	4,5	22991	0.91	4G	+ 0.05	2I + 0.02 2H
27	4,5	23041	0.93	4G	+ 0.04	2I + 0.02 2H
28	6	23058	0.91	4G	+ 0.06	2I + 0.02 2H
29	6	23290	0.36	4G	+ 0.28	2I + 0.14 2F1
30	4,5	23349	0.41	4G	+ 0.27	2I + 0.13 2F1
31	4,5	23450	0.35	2I	+ 0.28	4G + 0.16 2F1
32	4,5	24517	0.35	4G	+ 0.33	4D + 0.11 2I
33	6	24551	0.33	4G	+ 0.33	4D + 0.11 2I
34	4,5	24709	0.48	4G	+ 0.47	4D + 0.02 4F
35	4,5	24813	0.50	4D	+ 0.47	4G + 0.02 4F
36	6	24946	0.31	4D	+ 0.28	4G + 0.14 2I
37	4,5	25073	0.24	4D	+ 0.22	4G + 0.19 2I
38	4,5	25211	0.48	4D	+ 0.35	4G + 0.06 2I
39	6	25225	0.40	4D	+ 0.30	4G + 0.10 2I
40	4,5	26932	0.97	4D	+ 0.01	4F + 0.00 4G
41	6	26943	0.97	4D	+ 0.01	4F + 0.01 4G
42	4,5	26945	0.98	4D	+ 0.01	4F + 0.00 4G
43	4,5	26955	0.98	4D	+ 0.01	4G + 0.01 4F
44	6	28169	0.33	2G2	+ 0.20	2F2 + 0.18 2F
45	4,5	28201	0.31	2G2	+ 0.21	2F2 + 0.19 2H
46	4,5	28922	0.33	2G2	+ 0.20	2F2 + 0.17 2F1
47	4,5	29425	0.24	2G2	+ 0.21	2F2 + 0.15 2I
48	6	30528	0.30	2F2	+ 0.26	2G2 + 0.15 2D2
49	4,5	30642	0.28	2F2	+ 0.27	2G2 + 0.15 2D2
50	4,5	31183	0.32	2G2	+ 0.32	2I + 0.21 2G1
51	6	33547	0.39	4F	+ 0.31	4P + 0.11 2I

TABLE 16. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tb}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F(2) = 52465$ ,  
 $F(4) = 43188$ ,  $\zeta = 370$ ,  $B_{20} = 4043.5$ , and  $B_{40} = -22668$   
 ( $Dq = 1557.62$ ,  $B = 581.05$ , and  $C = 3427.62$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
52	4,5	33604	0.43	4F	+	0.35	4P	+	0.08	2I
53	4,5	34034	0.32	2I	+	0.16	2D3	+	0.16	4F
54	6	34182	0.29	2I	+	0.19	4F	+	0.15	2D3
55	4,5	34718	0.41	4F	+	0.23	4P	+	0.20	2I
56	4,5	34792	0.37	4F	+	0.23	2I	+	0.21	4P
57	6	34889	0.50	4F	+	0.28	4P	+	0.06	4G
58	4,5	34990	0.53	4F	+	0.29	4P	+	0.07	4G
59	4,5	35161	0.31	4F	+	0.26	2I	+	0.18	4P
60	6	35781	0.52	2I	+	0.31	2D3	+	0.06	2D1
61	4,5	35783	0.51	2I	+	0.31	2D3	+	0.06	2D1
62	4,5	35851	0.51	2I	+	0.35	2D3	+	0.07	2D1
63	4,5	36850	0.97	4F	+	0.01	2D3	+	0.01	2I
64	6	36857	0.97	4F	+	0.01	2D3	+	0.01	2I
65	6	38123	0.67	2G2	+	0.12	2I	+	0.06	2D2
66	4,5	38200	0.68	2G2	+	0.13	2I	+	0.06	2D2
67	6	395001	0.35	2F2	+	0.28	2G2	+	0.15	2I
68	4,5	39583	0.37	2F2	+	0.27	2G2	+	0.14	2I
69	4,5	40111	0.36	2F2	+	0.27	2H	+	0.14	2G2
70	4,5	40180	0.55	2F2	+	0.19	2I	+	0.18	2G2
71	4,5	40296	0.47	2F2	+	0.30	2G2	+	0.13	2I
72	6	40827	0.43	2F2	+	0.31	2G2	+	0.13	2H
73	4,5	40912	0.49	2F2	+	0.38	2G2	+	0.06	2I
74	4,5	41544	0.56	2H	+	0.28	2F1	+	0.06	2G2
75	6	41693	0.48	2H	+	0.24	2F1	+	0.13	2G2
76	4,5	42131	0.33	2H	+	0.28	2F2	+	0.15	2F1
77	4,5	42629	0.31	4F	+	0.28	4G	+	0.24	4P
78	4,5	42811	0.33	4F	+	0.30	4G	+	0.26	4P
79	6	42864	0.36	4F	+	0.32	4G	+	0.28	4P
80	4,5	42970	0.38	4F	+	0.32	4G	+	0.28	4P
81	4,5	43631	0.54	2S	+	0.26	2G2	+	0.05	4F
82	6	44285	0.41	4F	+	0.31	4G	+	0.22	4P
83	4,5	44353	0.41	4F	+	0.31	4G	+	0.22	4P
84	6	45003	0.35	2D3	+	0.31	2H	+	0.21	2D2
85	4,5	45038	0.36	2D3	+	0.31	2H	+	0.22	2D2
86	4,5	46634	0.36	2H	+	0.22	2F1	+	0.17	2G1
87	4,5	47556	0.64	4F	+	0.21	4D	+	0.11	4G
88	4,5	47634	0.61	4F	+	0.20	4D	+	0.10	4G
89	6	47704	0.54	4F	+	0.18	4D	+	0.09	4G

TABLE 16. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tb}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F(2) = 52465$ ,  
 $F(4) = 43188$ ,  $\zeta = 370$ ,  $B_{20} = 4043.5$ , and  $B_{40} = -22668$   
 ( $Dq = 1557.62$ ,  $B = 581.05$ , and  $C = 3427.62$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
90	4,5	47741	0.30	4F	+	0.22	2H	+	0.11	4D
91	6	48035	0.32	2H	+	0.13	2F1	+	0.13	2G1
92	4,5	48122	0.36	4F	+	0.17	2H	+	0.13	4D
93	6	48157	0.57	4F	+	0.18	4D	+	0.10	4G
94	4,5	48208	0.63	4F	+	0.20	4D	+	0.12	4G
95	4,5	48839	0.26	2F1	+	0.17	2H	+	0.16	2D2
96	6	48995	0.25	2F1	+	0.16	2H	+	0.15	2D2
97	4,5	49504	0.28	2F1	+	0.17	2H	+	0.16	2G1
98	4,5	52981	0.27	2G1	+	0.27	2H	+	0.19	2I
99	6	53066	0.30	2G1	+	0.26	2H	+	0.18	2I
100	4,5	53877	0.36	2G1	+	0.18	2H	+	0.16	2I
101	4,5	53920	0.39	2F1	+	0.24	2I	+	0.13	2F2
102	6	54041	0.46	2G1	+	0.22	2H	+	0.14	2I
103	4,5	54662	0.35	2H	+	0.25	2I	+	0.21	2F1
104	4,5	57430	0.48	2D2	+	0.14	2I	+	0.12	2G1
105	6	57742	0.38	2D2	+	0.20	2G1	+	0.15	2I
106	4,5	57790	0.38	2D2	+	0.20	2G1	+	0.15	2I
107	4,5	59189	0.35	2G1	+	0.23	2D2	+	0.11	2H
108	6	59230	0.41	2G1	+	0.17	2D2	+	0.10	2H
109	6	60087	0.32	2D2	+	0.24	2G1	+	0.11	2H
110	4,5	60129	0.31	2D2	+	0.27	2G1	+	0.09	2H
111	4,5	60362	0.67	2G1	+	0.15	2P	+	0.09	2F2
112	4,5	61560	0.25	2H	+	0.24	2D3	+	0.15	2I
113	6	62103	0.27	2D3	+	0.16	2H	+	0.15	2F1
114	4,5	62229	0.26	2D3	+	0.16	2H	+	0.14	2F1
115	4,5	63252	0.31	2G1	+	0.22	2D1	+	0.15	2G2
116	6	64116	0.47	2G1	+	0.14	2G2	+	0.13	2F2
117	4,5	64227	0.45	2G1	+	0.15	2G2	+	0.13	2D1
118	4,5	66778	0.69	2G1	+	0.17	2S	+	0.12	2G2
119	4,5	69424	0.78	2P	+	0.07	2G2	+	0.07	2G1
120	4,5	69747	0.75	2P	+	0.07	2F2	+	0.06	2G2
121	6	69762	0.76	2P	+	0.07	2F2	+	0.06	2G2
122	6	82044	0.64	2D1	+	0.08	2G1	+	0.08	2H
123	4,5	82154	0.65	2D1	+	0.11	2G1	+	0.08	2D3
124	6	82156	0.65	2D1	+	0.11	2G1	+	0.08	2D3
125	4,5	82231	0.64	2D1	+	0.08	2G1	+	0.08	2H
126	4,5	84509	0.56	2D1	+	0.11	2G1	+	0.09	2H

<sup>a</sup>Irreducible representation of the  $C_{2v}$  group.

TABLE 17. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Dy}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22745$   
 $(Dq = 1556.23, B = 588.45, \text{ and } C = 3413.41)$ . All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.140	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.255	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	8449	0.25	2I	+	0.22	2H	+	0.16	2F1
5	4,5	9185	0.24	2I	+	0.22	2H	+	0.14	2F1
6	6	9435	0.24	4G	+	0.18	2I	+	0.16	2H
7	6	9896	0.50	4G	+	0.29	4P	+	0.05	4F
8	4,5	9910	0.55	4G	+	0.31	4P	+	0.06	4F
9	4,5	10225	0.56	4G	+	0.35	4P	+	0.04	4F
10	6	10348	0.43	4G	+	0.27	4P	+	0.08	2I
11	4,5	10367	0.47	4G	+	0.30	4P	+	0.06	2I
12	4,5	10389	0.56	4G	+	0.35	4P	+	0.04	4F
13	4,5	13344	0.45	4G	+	0.32	4F	+	0.23	4D
14	6	13364	0.45	4G	+	0.32	4F	+	0.22	4D
15	4,5	14602	0.42	4G	+	0.30	4F	+	0.27	4D
16	4,5	14610	0.43	4G	+	0.30	4F	+	0.26	4D
17	6	14611	0.42	4G	+	0.30	4F	+	0.27	4D
18	4,5	14620	0.42	4G	+	0.30	4F	+	0.27	4D
19	4,5	21258	0.52	2I	+	0.25	2H	+	0.13	2F1
20	4,5	21550	0.52	2I	+	0.40	2F1	+	0.05	2F2
21	6	21559	0.53	2I	+	0.23	2H	+	0.12	2F1
22	4,5	218901	0.53	2I	+	0.22	2H	+	0.12	2F1
23	4,5	22488	0.50	4G	+	0.20	2I	+	0.11	2F1
24	6	22672	0.51	4G	+	0.21	2I	+	0.11	2F1
25	4,5	22699	0.72	4G	+	0.12	2I	+	0.07	2F1
26	4,5	22938	0.91	4G	+	0.05	2I	+	0.02	2H
27	4,5	23030	0.93	4G	+	0.04	2I	+	0.01	2H
28	6	23037	0.89	4G	+	0.06	2I	+	0.02	2H
29	6	23188	0.51	4G	+	0.21	2I	+	0.10	2F1
30	4,5	23263	0.50	4G	+	0.22	2I	+	0.10	2F1
31	4,5	23407	0.36	2I	+	0.26	4G	+	0.17	2F1
32	4,5	24508	0.38	4G	+	0.35	4D	+	0.09	2I
33	6	24548	0.37	4G	+	0.36	4D	+	0.09	2I
34	4,5	24677	0.48	4G	+	0.46	4D	+	0.02	4F
35	4,5	24772	0.49	4D	+	0.47	4G	+	0.03	4F
36	6	24966	0.24	4D	+	0.22	4G	+	0.19	2I
37	4,5	25081	0.23	2I	+	0.18	4D	+	0.18	2H
38	4,5	25283	0.52	4D	+	0.36	4G	+	0.04	2I

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 17. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Dy}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22745$   
 $(Dq = 1556.23, B = 588.45, \text{ and } C = 3413.41)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
39	6	25294	0.45	4D	+	0.32	4G	+	0.08	2I
40	4,5	26921	0.96	4D	+	0.01	4F	+	0.01	4G
41	6	26935	0.96	4D	+	0.01	4F	+	0.01	4G
42	4,5	26937	0.97	4D	+	0.01	4F	+	0.01	4G
43	4,5	26949	0.97	4D	+	0.01	4F	+	0.01	4G
44	6	28111	0.33	2G2	+	0.19	2F2	+	0.18	2F1
45	4,5	28147	0.31	2G2	+	0.20	2F2	+	0.19	2H
46	4,5	29047	0.31	2G2	+	0.20	2F2	+	0.12	2H
47	4,5	29263	0.27	2G2	+	0.19	2F2	+	0.13	2I
48	6	30697	0.30	2F2	+	0.26	2G2	+	0.15	2D2
49	4,5	30793	0.27	2F2	+	0.27	2G2	+	0.14	2D2
50	4,5	31260	0.32	2I	+	0.32	2G2	+	0.21	2G1
51	6	33422	0.40	4F	+	0.35	4P	+	0.09	2I
52	4,5	33469	0.44	4F	+	0.39	4P	+	0.06	4G
53	4,5	34009	0.37	2I	+	0.19	2D3	+	0.11	2D1
54	6	34133	0.34	2I	+	0.18	2D3	+	0.13	4F
55	4,5	34809	0.49	2I	+	0.17	2G2	+	0.15	4F
56	4,5	34948	0.48	4F	+	0.28	4P	+	0.08	2I
57	6	35072	0.50	4F	+	0.29	4P	+	0.07	4G
58	4,5	35194	0.53	4F	+	0.30	4P	+	0.08	4G
59	4,5	35319	0.44	4F	+	0.25	4P	+	0.11	2I
60	6	35725	0.50	2I	+	0.30	2D3	+	0.06	2D1
61	4,5	35735	0.48	2I	+	0.29	2D3	+	0.06	4F
62	4,5	35819	0.50	2I	+	0.35	2D3	+	0.07	2D1
63	4,5	36911	0.98	4F	+	0.01	2D3	+	0.01	2I
64	6	36919	0.98	4F	+	0.01	2D3	+	0.01	2I
65	6	38021	0.65	2G2	+	0.11	2I	+	0.06	2H
66	4,5	38125	0.67	2G2	+	0.12	2I	+	0.06	2D2
67	6	39391	0.34	2F2	+	0.27	2G2	+	0.15	2I
68	4,5	39462	0.36	2F2	+	0.25	2G2	+	0.14	2I
69	4,5	39822	0.36	2F2	+	0.30	2H	+	0.16	2F1
70	4,5	40070	0.52	2F2	+	0.21	2G2	+	0.21	2I
71	4,5	40420	0.48	2F2	+	0.31	2G2	+	0.12	2I
72	6	40858	0.42	2F2	+	0.31	2G2	+	0.14	2H
73	4,5	40961	0.47	2F2	+	0.38	2G2	+	0.06	2I
74	4,5	41635	0.50	2H	+	0.27	2F1	+	0.09	2G2
75	6	41803	0.44	2H	+	0.23	2F1	+	0.15	2G2
76	4,5	42338	0.19	4F	+	0.19	2F2	+	0.16	4G



TABLE 17. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Dy}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22745$   
 ( $Dq = 1556.23$ ,  $B = 588.45$ , and  $C = 3413.41$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
77	4,5	42543	0.30	4F	+	0.25	4G	+	0.22	4P
78	6	42711	0.37	4F	+	0.31	4G	+	0.27	4P
79	4,5	42739	0.23	4F	+	0.21	2H	+	0.20	4G
80	4,5	42814	0.38	4F	+	0.31	4G	+	0.27	4P
81	4,5	43599	0.56	2S	+	0.27	2G2	+	0.04	4F
82	6	44568	0.39	4F	+	0.29	4G	+	0.19	4P
83	4,5	44638	0.41	4F	+	0.30	4G	+	0.21	4P
84	6	44949	0.30	2D3	+	0.30	2H	+	0.17	2D2
85	4,5	44982	0.33	2D3	+	0.31	2H	+	0.19	2D2
86	4,5	46476	0.36	2H	+	0.23	2F1	+	0.17	2G1
87	4,5	47655	0.64	4F	+	0.21	4D	+	0.11	4G
88	4,5	47746	0.61	4F	+	0.20	4D	+	0.10	4G
89	6	47833	0.55	4F	+	0.19	4D	+	0.09	4G
90	4,5	47913	0.39	4F	+	0.17	2H	+	0.14	4D
91	6	48236	0.26	4F	+	0.21	2H	+	0.11	2F1
92	4,5	48303	0.31	4F	+	0.20	2H	+	0.11	4D
93	6	48357	0.35	4F	+	0.17	2H	+	0.11	4D
94	4,5	48394	0.54	4F	+	0.17	4D	+	0.11	4G
95	4,5	48845	0.23	2F1	+	0.17	2H	+	0.14	2D2
96	6	49022	0.22	2F1	+	0.15	2H	+	0.15	4F
97	4,5	49659	0.28	2F1	+	0.17	2H	+	0.15	2G1
98	4,5	52840	0.28	2H	+	0.22	2G1	+	0.21	2I
99	6	52927	0.27	2H	+	0.23	2G1	+	0.20	2I
100	4,5	53818	0.47	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	53986	0.50	2G1	+	0.20	2H	+	0.12	2I
102	6	54156	0.51	2G1	+	0.20	2H	+	0.12	2I
103	4,5	54910	0.36	2H	+	0.25	2I	+	0.20	2F1
104	4,5	57376	0.50	2D2	+	0.13	2I	+	0.13	2G1
105	6	57665	0.39	2D2	+	0.22	2G1	+	0.14	2I
106	4,5	57700	0.39	2D2	+	0.23	2G1	+	0.14	2I
107	4,5	59133	0.35	2G1	+	0.20	2D2	+	0.11	2H
108	6	59188	0.39	2G1	+	0.17	2D2	+	0.11	2H
109	6	60423	0.32	2D2	+	0.23	2G1	+	0.11	2H
110	4,5	60463	0.33	2D2	+	0.24	2G1	+	0.09	2H
111	4,5	60686	0.67	2G1	+	0.15	2P	+	0.09	2F2
112	4,5	61587	0.26	2H	+	0.22	2D3	+	0.16	2G1
113	6	62295	0.25	2D3	+	0.16	2D2	+	0.16	2H
114	4,5	62410	0.24	2D3	+	0.16	2H	+	0.15	2G1
115	4,5	63258	0.27	2G1	+	0.23	2D1	+	0.15	2G2

TABLE 17. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Dy}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22745$   
 ( $Dq = 1556.23$ ,  $B = 588.45$ , and  $C = 3413.41$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition
116	6	64354	0.48 2G1 + 0.14 2G2 + 0.13 2F2
117	4,5	64482	0.45 2G1 + 0.15 2G2 + 0.11 2D1
118	4,5	66958	0.69 2G1 + 0.17 2S + 0.12 2G2
119	4,5	69574	0.78 2P + 0.07 2G2 + 0.07 2G1
120	4,5	69846	0.73 2P + 0.07 2F2 + 0.06 2G2
121	6	69876	0.74 2P + 0.07 2F2 + 0.06 2G2
122	6	82094	0.63 2D1 + 0.11 2H + 0.05 2G1
123	4,5	82196	0.64 2D1 + 0.10 2H + 0.06 2G1
124	6	82532	0.64 2D1 + 0.13 2G1 + 0.11 2D3
125	4,5	82651	0.64 2D1 + 0.12 2G1 + 0.10 2D3
126	4,5	85155	0.55 2D1 + 0.12 2G1 + 0.09 2H

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 18. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Ho}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22822$   
 ( $Dq = 1554.85$ ,  $B = 595.85$ , and  $C = 3399.21$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition
1	4,5	0.000	1.00 6S + 0.00 4P + 0.00 4G
2	6	0.145	1.00 6S + 0.00 4P + 0.00 4G
3	4,5	0.264	1.00 6S + 0.00 4P + 0.00 4G
4	4,5	8262	0.25 2I + 0.23 2H + 0.15 2F1
5	4,5	9118	0.24 2I + 0.22 2H + 0.14 2F1
6	6	9371	0.31 4G + 0.17 4P + 0.15 2I
7	4,5	9758	0.55 4G + 0.31 4P + 0.06 4F
8	6	9793	0.45 4G + 0.26 4P + 0.08 2I
9	4,5	10164	0.54 4G + 0.35 4P + 0.05 4F
10	6	10288	0.41 4G + 0.26 4P + 0.09 2I
11	4,5	10301	0.46 4G + 0.29 4P + 0.06 2I
12	4,5	10337	0.55 4G + 0.35 4P + 0.04 4F
13	4,5	13153	0.46 4G + 0.32 4F + 0.22 4D
14	6	13175	0.46 4G + 0.32 4F + 0.22 4D
15	4,5	14748	0.43 4G + 0.30 4F + 0.26 4D

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 18. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Ho}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F(2) = 52728$ ,  
 $F(4) = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22822$   
 $(Dq = 1554.85, B = 595.85, \text{ and } C = 3399.21)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
16	4,5	14755	0.43	4G	+	0.29	4F	+	0.26	4D
17	6	14763	0.42	4G	+	0.29	4F	+	0.26	4D
18	4,5	14774	0.42	4G	+	0.29	4F	+	0.27	4D
19	4,5	21102	0.52	2I	+	0.25	2H	+	0.13	2F1
20	4,5	21464	0.52	2I	+	0.40	2F1	+	0.05	2F2
21	6	21473	0.53	2I	+	0.23	2H	+	0.12	2F1
22	4,5	21798	0.53	2I	+	0.22	2H	+	0.12	2F1
23	4,5	22374	0.42	4G	+	0.23	2I	+	0.13	2F1
24	6	22546	0.37	4G	+	0.26	2I	+	0.13	2F1
25	4,5	22628	0.72	4G	+	0.12	2I	+	0.06	2F1
26	4,5	22867	0.90	4G	+	0.04	2I	+	0.02	2H
27	6	22981	0.81	4G	+	0.08	2I	+	0.03	2F1
28	4,5	23011	0.91	4G	+	0.04	2I	+	0.02	2H
29	6	23117	0.73	4G	+	0.12	2I	+	0.05	2F1
30	4,5	23178	0.61	4G	+	0.17	2I	+	0.07	2F1
31	4,5	23349	0.37	2I	+	0.25	4G	+	0.17	2F1
32	4,5	24489	0.41	4G	+	0.37	4D	+	0.07	2I
33	6	24530	0.41	4G	+	0.38	4D	+	0.07	2I
34	4,5	24634	0.48	4G	+	0.46	4D	+	0.03	4F
35	4,5	24721	0.48	4D	+	0.48	4G	+	0.03	4F
36	6	24992	0.24	2I	+	0.19	4D	+	0.17	2H
37	4,5	25093	0.26	2I	+	0.19	2H	+	0.14	4D
38	4,5	25361	0.54	4D	+	0.36	4G	+	0.03	2I
39	6	25370	0.48	4D	+	0.32	4G	+	0.07	2I
40	4,5	26897	0.95	4D	+	0.02	4F	+	0.01	4G
41	6	26914	0.95	4D	+	0.02	4F	+	0.01	4G
42	4,5	26915	0.96	4D	+	0.02	4F	+	0.01	4G
43	4,5	26929	0.96	4D	+	0.01	4F	+	0.01	4G
44	6	28050	0.32	2G2	+	0.19	2F2	+	0.18	2H
45	4,5	28090	0.30	2G2	+	0.20	2F2	+	0.19	2H
46	4,5	28937	0.24	2G2	+	0.19	2F2	+	0.15	2I
47	4,5	29320	0.33	2G2	+	0.19	2F2	+	0.16	2F1
48	6	30864	0.30	2F2	+	0.27	2G2	+	0.15	2D2
49	4,5	30939	0.27	2G2	+	0.27	2F2	+	0.14	2D2
50	4,5	31338	0.33	2I	+	0.32	2G2	+	0.20	2G1
51	6	33288	0.41	4F	+	0.38	4P	+	0.07	2I
52	4,5	33325	0.44	4F	+	0.42	4P	+	0.07	4G
53	4,5	33978	0.40	2I	+	0.20	2D3	+	0.11	2D1

TABLE 18. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Ho}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22822$   
 ( $Dq = 1554.85$ ,  $B = 595.85$ , and  $C = 3399.21$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
54	6	34079	0.37	2I	+	0.19	2D3	+	0.11	2D1
55	4,5	34831	0.56	2I	+	0.21	2G2	+	0.06	4F
56	4,5	35145	0.44	4F	+	0.26	4P	+	0.10	2I
57	6	35274	0.44	4F	+	0.26	4P	+	0.10	2I
58	4,5	35423	0.50	4F	+	0.29	4P	+	0.08	4G
59	4,5	35543	0.48	4F	+	0.28	4P	+	0.09	4G
60	6	35676	0.44	2I	+	0.26	2D3	+	0.11	4F
61	4,5	35701	0.41	2I	+	0.25	2D3	+	0.14	4F
62	4,5	35787	0.49	2I	+	0.34	2D3	+	0.07	2D1
63	4,5	36972	0.98	4F	+	0.01	2D3	+	0.01	2I
64	6	36981	0.98	4F	+	0.01	2D3	+	0.01	2I
65	6	37892	0.63	2G2	+	0.10	2I	+	0.08	2H
66	4,5	38023	0.65	2G2	+	0.11	2I	+	0.06	2D2
67	6	39266	0.33	2F2	+	0.26	2G2	+	0.16	2I
68	4,5	39323	0.35	2F2	+	0.23	2G2	+	0.15	2I
69	4,5	39515	0.35	2F2	+	0.31	2H	+	0.17	2F1
70	4,5	39945	0.51	2F2	+	0.23	2I	+	0.21	2G2
71	4,5	40554	0.48	2F2	+	0.31	2G2	+	0.11	2I
72	6	40897	0.41	2F2	+	0.31	2G2	+	0.13	2H
73	4,5	41010	0.46	2F2	+	0.38	2G2	+	0.06	2H
74	4,5	41740	0.45	2H	+	0.27	2F1	+	0.11	2G2
75	6	41914	0.40	2H	+	0.22	2F1	+	0.17	2G2
76	4,5	42313	0.34	4F	+	0.28	4G	+	0.24	4P
77	4,5	42451	0.35	4F	+	0.28	4G	+	0.25	4P
78	6	42558	0.38	4F	+	0.30	4G	+	0.26	4P
79	4,5	42648	0.39	4F	+	0.31	4G	+	0.27	4P
80	4,5	42907	0.38	2H	+	0.27	2F2	+	0.13	2F1
81	4,5	43541	0.56	2S	+	0.27	2G2	+	0.04	2G1
82	6	44736	0.25	2D3	+	0.23	2H	+	0.15	2D2
83	4,5	44835	0.26	2D3	+	0.24	2H	+	0.15	2D2
84	6	45005	0.32	4F	+	0.24	4G	+	0.16	4P
85	4,5	45011	0.33	4F	+	0.25	4G	+	0.16	4P
86	4,5	46319	0.35	2H	+	0.24	2F1	+	0.17	2G1
87	4,5	47780	0.63	4F	+	0.22	4D	+	0.11	4G
88	4,5	47878	0.59	4F	+	0.20	4D	+	0.10	4G
89	6	47975	0.53	4F	+	0.18	4D	+	0.09	4G
90	4,5	48086	0.42	4F	+	0.16	4D	+	0.15	2H
91	6	48365	0.31	4F	+	0.15	2H	+	0.11	2F1

TABLE 18. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Ho}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 52728$ ,  
 $F^{(4)} = 43009$ ,  $\zeta = 370$ ,  $B_{20} = 5143.7$ , and  $B_{40} = -22822$   
 ( $Dq = 1554.85$ ,  $B = 595.85$ , and  $C = 3399.21$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
92	4,5	48428	0.31	4F	+	0.15	2H	+	0.11	2F1
93	6	48616	0.25	4F	+	0.25	2H	+	0.09	2F2
94	4,5	48623	0.44	4F	+	0.14	4D	+	0.13	2H
95	4,5	48925	0.18	4F	+	0.18	2F1	+	0.17	2H
96	6	49099	0.21	4F	+	0.18	2F1	+	0.15	2H
97	4,5	49821	0.29	2F1	+	0.18	2H	+	0.14	2G1
98	4,5	52704	0.28	2H	+	0.22	2I	+	0.19	2F1
99	6	52788	0.28	2H	+	0.22	2I	+	0.19	2G1
100	4,5	53723	0.47	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	54077	0.52	2G1	+	0.19	2H	+	0.11	2I
102	6	54269	0.53	2G1	+	0.19	2H	+	0.12	2I
103	4,5	55174	0.36	2H	+	0.24	2I	+	0.19	2F1
104	4,5	57343	0.52	2D2	+	0.14	2G1	+	0.12	2I
105	6	57587	0.40	2D2	+	0.24	2G1	+	0.13	2I
106	4,5	57606	0.40	2D2	+	0.25	2G1	+	0.13	2I
107	4,5	59073	0.36	2G1	+	0.18	2D2	+	0.11	2H
108	6	59140	0.38	2G1	+	0.16	2D2	+	0.11	2H
109	6	60784	0.32	2D2	+	0.22	2G1	+	0.10	2H
110	4,5	60819	0.35	2D2	+	0.21	2G1	+	0.09	2I
111	4,5	61024	0.66	2G1	+	0.15	2P	+	0.10	2F2
112	4,5	61617	0.27	2H	+	0.19	2D3	+	0.19	2G1
113	6	62513	0.24	2D3	+	0.17	2D2	+	0.15	2H
114	4,5	62616	0.22	2D3	+	0.18	2G1	+	0.16	2H
115	4,5	63276	0.23	2D1	+	0.23	2G1	+	0.16	2D3
116	6	64597	0.48	2G1	+	0.14	2G2	+	0.12	2F2
117	4,5	64744	0.46	2G1	+	0.15	2G2	+	0.10	2D1
118	4,5	67154	0.68	2G1	+	0.17	2S	+	0.12	2G2
119	4,5	69729	0.77	2P	+	0.07	2G2	+	0.07	2G1
120	4,5	69919	0.73	2P	+	0.07	2F2	+	0.06	2G2
121	6	69952	0.73	2P	+	0.07	2F2	+	0.06	2D1
122	6	82122	0.64	2D1	+	0.10	2H	+	0.05	2G1
123	4,5	82235	0.64	2D1	+	0.10	2H	+	0.06	2G1
124	6	83035	0.63	2D1	+	0.13	2G1	+	0.11	2D3
125	4,5	83177	0.62	2D1	+	0.12	2G1	+	0.11	2D3
126	4,5	858278	0.54	2D1	+	0.13	2G1	+	0.08	2H

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 19. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Er}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F(2) = 53254$ ,  
 $F(4) = 42651$ ,  $\zeta = 370$ ,  $B_{20} = 7344.2$ , and  $B_{40} = -22898$   
 ( $Dq = 1553.46$ ,  $B = 603.24$ , and  $C = 3385.00$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.174	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.313	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	8050	0.25	2I	+	0.23	2H	+	0.15	2F1
5	4,5	9047	0.24	2I	+	0.21	2H	+	0.14	2F1
6	6	9278	0.37	4G	+	0.21	4P	+	0.11	2I
7	4,5	9598	0.56	4G	+	0.31	4P	+	0.07	4F
8	6	9693	0.39	4G	+	0.23	4P	+	0.10	2I
9	4,5	10085	0.53	4G	+	0.34	4P	+	0.05	4F
10	6	10214	0.38	4G	+	0.24	4P	+	0.10	2I
11	4,5	10217	0.45	4G	+	0.29	4P	+	0.06	2I
12	4,5	10266	0.55	4G	+	0.35	4P	+	0.04	4F
13	4,5	12951	0.47	4G	+	0.32	4F	+	0.21	4D
14	6	12975	0.47	4G	+	0.32	4F	+	0.21	4D
15	4,5	14895	0.44	4G	+	0.29	4F	+	0.25	4D
16	4,5	14906	0.43	4G	+	0.29	4F	+	0.26	4D
17	6	14920	0.43	4G	+	0.29	4F	+	0.26	4D
18	4,5	14936	0.42	4G	+	0.29	4F	+	0.26	4D
19	4,5	20933	0.52	2I	+	0.26	2H	+	0.13	2F1
20	6	21365	0.53	2I	+	0.23	2H	+	0.11	2F1
21	4,5	21367	0.52	2I	+	0.40	2F1	+	0.06	2F2
22	4,5	21674	0.52	2I	+	0.22	2H	+	0.12	2F1
23	4,5	22238	0.35	4G	+	0.26	2I	+	0.14	2F1
24	6	22383	0.30	2I	+	0.25	4G	+	0.15	2F1
25	4,5	22540	0.72	4G	+	0.12	2I	+	0.06	2F1
26	4,5	22777	0.90	4G	+	0.04	2I	+	0.02	4F
27	6	22884	0.79	4G	+	0.08	2I	+	0.03	2F1
28	4,5	22970	0.84	4G	+	0.06	2I	+	0.03	2F1
29	6	23088	0.86	4G	+	0.07	2I	+	0.03	2H
30	4,5	23109	0.76	4G	+	0.11	2I	+	0.04	2F1
31	4,5	23274	0.38	2I	+	0.23	4G	+	0.18	2F1
32	4,5	24458	0.44	4G	+	0.38	4D	+	0.06	2I
33	6	24498	0.44	4G	+	0.40	4D	+	0.05	2I
34	4,5	24583	0.49	4G	+	0.45	4D	+	0.03	4F
35	4,5	24663	0.48	4G	+	0.47	4D	+	0.03	4F
36	6	25024	0.27	2I	+	0.18	2H	+	0.15	4D
37	4,5	25108	0.28	2I	+	0.20	2H	+	0.11	4G
38	4,5	25443	0.56	4D	+	0.35	4G	+	0.03	2I

<sup>a</sup>Irreducible representation of the cubic group.

TABLE 19. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Er}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F(2) = 53254$ ,  
 $F(4) = 42651$ ,  $\zeta = 370$ ,  $B_{20} = 7344.2$ , and  $B_{40} = -22898$   
 ( $Dq = 1553.46$ ,  $B = 603.24$ , and  $C = 3385.00$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>d</sup>	Energy	Free ion state composition							
39	6	25451	0.51	4D	+	0.32	4G	+	0.06	2I
40	4,5	26862	0.94	4D	+	0.02	4F	+	0.02	4G
41	6	26881	0.94	4D	+	0.02	4G	+	0.02	4F
42	4,5	26883	0.95	4D	+	0.02	4F	+	0.02	4G
43	4,5	26896	0.94	4D	+	0.02	4G	+	0.02	4F
44	6	27988	0.32	2G2	+	0.19	2F2	+	0.19	2H
45	4,5	28031	0.30	2G2	+	0.19	2F2	+	0.19	2H
46	4,5	28714	0.23	2G2	+	0.17	2I	+	0.17	2F2
47	4,5	29477	0.34	2G2	+	0.19	2F2	+	0.17	2F1
48	6	31028	0.29	2F2	+	0.27	2G2	+	0.14	2D2
49	4,5	31081	0.27	2G2	+	0.26	2F2	+	0.14	2I
50	4,5	31416	0.33	2I	+	0.32	2G2	+	0.19	2G1
51	6	33147	0.41	4P	+	0.41	4F	+	0.06	4G
52	4,5	33176	0.44	4P	+	0.43	4F	+	0.07	4G
53	4,5	33942	0.42	2I	+	0.20	2D3	+	0.11	2D1
54	6	34021	0.39	2I	+	0.20	2D3	+	0.11	2D1
55	4,5	34823	0.58	2I	+	0.23	2G2	+	0.05	2G1
56	4,5	35304	0.26	2I	+	0.25	4F	+	0.16	4P
57	6	35404	0.34	2I	+	0.19	2D3	+	0.19	4F
58	4,5	35630	0.40	4F	+	0.23	4P	+	0.14	2I
59	6	35719	0.35	4F	+	0.19	2I	+	0.19	4P
60	4,5	35724	0.43	2I	+	0.29	2D3	+	0.07	4F
61	4,5	35779	0.40	4F	+	0.23	4P	+	0.13	2I
62	4,5	35844	0.44	4F	+	0.26	4P	+	0.09	2I
63	4,5	37033	0.98	4F	+	0.01	2D3	+	0.01	2I
64	6	37044	0.98	4F	+	0.01	2D3	+	0.00	2I
65	6	37739	0.61	2G2	+	0.10	2H	+	0.09	2I
66	4,5	37895	0.64	2G2	+	0.10	2I	+	0.08	2H
67	6	39134	0.33	2F2	+	0.25	2G2	+	0.17	2I
68	4,5	39155	0.35	2F2	+	0.21	2H	+	0.16	2G2
69	4,5	39217	0.34	2F2	+	0.23	2H	+	0.15	2G2
70	4,5	39813	0.49	2F2	+	0.25	2I	+	0.20	2G2
71	4,5	40688	0.49	2F2	+	0.31	2G2	+	0.11	2I
72	6	40942	0.40	2F2	+	0.32	2G2	+	0.12	2H
73	4,5	41059	0.45	2F2	+	0.38	2G2	+	0.06	2H
74	4,5	41843	0.40	2H	+	0.25	2F1	+	0.13	2G2
75	6	42015	0.34	2H	+	0.22	2F1	+	0.17	2G2
76	4,5	42203	0.37	4F	+	0.28	4G	+	0.25	4P

TABLE 19. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Er}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 53254$ ,  
 $F^{(4)} = 42651$ ,  $\zeta = 370$ ,  $B_{20} = 7344.2$ , and  $B_{40} = -22898$   
 ( $Dq = 1553.46$ ,  $B = 603.24$ , and  $C = 3385.00$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>d</sup>	Energy	Free ion state composition							
77	4,5	42326	0.37	4F	+	0.28	4G	+	0.24	4P
78	6	42410	0.37	4F	+	0.28	4G	+	0.24	4P
79	4,5	42489	0.41	4F	+	0.30	4G	+	0.26	4P
80	4,5	43190	0.39	2H	+	0.32	2F2	+	0.14	2F1
81	4,5	43461	0.55	2S	+	0.27	2G2	+	0.04	2G1
82	6	44680	0.32	2H	+	0.31	2D3	+	0.18	2D2
83	4,5	44772	0.33	2H	+	0.33	2D3	+	0.18	2D2
84	6	45285	0.44	4F	+	0.31	4G	+	0.20	4P
85	4,5	45302	0.45	4F	+	0.32	4G	+	0.20	4P
86	4,5	46163	0.33	2H	+	0.25	2F1	+	0.17	2G1
87	4,5	47929	0.63	4F	+	0.22	4D	+	0.10	4G
88	4,5	48022	0.53	4F	+	0.18	4D	+	0.09	4G
89	6	48120	0.46	4F	+	0.16	4D	+	0.08	4G
90	4,5	48246	0.38	4F	+	0.16	2H	+	0.14	4D
91	6	48449	0.31	4F	+	0.14	2H	+	0.12	2F1
92	4,5	48498	0.36	4F	+	0.13	4D	+	0.11	2F1
93	4,5	48872	0.48	4F	+	0.14	4D	+	0.12	2H
94	6	48885	0.29	4F	+	0.23	2H	+	0.09	4D
95	4,5	49106	0.22	2H	+	0.19	4F	+	0.13	2F1
96	6	49247	0.23	4F	+	0.17	2H	+	0.14	2F1
97	4,5	49980	0.29	2F1	+	0.18	2H	+	0.13	2I
98	4,5	52577	0.28	2H	+	0.22	2I	+	0.20	2F1
99	6	52656	0.28	2H	+	0.23	2I	+	0.20	2F1
100	4,5	53620	0.47	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	54161	0.53	2G1	+	0.18	2H	+	0.11	2I
102	6	54373	0.54	2G1	+	0.19	2H	+	0.11	2I
103	4,5	55451	0.36	2H	+	0.23	2I	+	0.19	2F1
104	4,5	57339	0.53	2D2	+	0.15	2G1	+	0.11	2I
105	6	57513	0.43	2D2	+	0.25	2G1	+	0.12	2I
106	4,5	57515	0.42	2D2	+	0.26	2G1	+	0.12	2I
107	4,5	59017	0.37	2G1	+	0.16	2D2	+	0.11	2H
108	6	59094	0.39	2G1	+	0.14	2D2	+	0.12	2H
109	6	61161	0.32	2D2	+	0.21	2G1	+	0.10	2H
110	4,5	61189	0.37	2D2	+	0.19	2G1	+	0.09	2F2
111	4,5	61372	0.64	2G1	+	0.15	2P	+	0.10	2F2
112	4,5	61649	0.27	2H	+	0.23	2G1	+	0.16	2D3
113	6	62752	0.22	2D3	+	0.17	2D2	+	0.15	2H
114	4,5	62842	0.21	2D3	+	0.21	2G1	+	0.15	2H



TABLE 19. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Er}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F(2) = 53254$ ,  
 $F(4) = 42651$ ,  $\zeta = 370$ ,  $B_{20} = 7344.2$ , and  $B_{40} = -22898$   
 ( $Dq = 1553.46$ ,  $B = 603.24$ , and  $C = 3385.00$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition
115	4,5	63312	0.23 2D1 + 0.19 2G1 + 0.18 2D3
116	6	64840	0.49 2G1 + 0.14 2G2 + 0.11 2F2
117	4,5	65007	0.46 2G1 + 0.15 2G2 + 0.09 2F2
118	4,5	67365	0.68 2G1 + 0.16 2S + 0.12 2G2
119	4,5	69846	0.73 2P + 0.06 2F2 + 0.06 2G1
120	6	69997	0.71 2P + 0.08 2D1 + 0.07 2F2
121	4,5	70015	0.76 2P + 0.06 2G1 + 0.06 2G2
122	6	82172	0.64 2D1 + 0.10 2H + 0.05 2G1
123	4,5	82297	0.65 2D1 + 0.10 2H + 0.05 2F1
124	6	83613	0.61 2D1 + 0.13 2G1 + 0.11 2D3
125	4,5	83774	0.60 2D1 + 0.12 2G1 + 0.11 2D3
126	4,5	86522	0.53 2D1 + 0.14 2G1 + 0.08 2H

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 20. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tm}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F(2) = 53517$ ,  
 $F(4) = 42471$ ,  $\zeta = 370$ ,  $B_{20} = 8444.4$  and  $B_{40} = -22975$   
 ( $Dq = 1552.10$ ,  $B = 610.65$ , and  $C = 3370.71$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition
1	4,5	0.000	1.00 6S + 0.00 4P + 0.00 4G
2	6	0.191	1.00 6S + 0.00 4P + 0.00 4G
3	4,5	0.333	1.00 6S + 0.00 4P + 0.00 4G
4	4,5	7815	0.25 2I + 0.23 2H + 0.15 2F1
5	4,5	8967	0.23 2I + 0.21 2H + 0.14 2F1
6	6	9159	0.43 4G + 0.23 4P + 0.08 2I
7	4,5	9429	0.56 4G + 0.30 4P + 0.08 4F
8	6	9594	0.35 4G + 0.21 4P + 0.12 2I
9	4,5	9987	0.51 4G + 0.33 4P + 0.05 4F
10	4,5	10116	0.44 4G + 0.28 4P + 0.06 2I
11	6	10125	0.36 4G + 0.23 4P + 0.11 2I
12	4,5	10178	0.54 4G + 0.35 4P + 0.05 4F
13	4,5	12738	0.48 4G + 0.32 4F + 0.20 4D
14	6	12763	0.47 4G + 0.32 4F + 0.20 4D

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 20. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tm}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 53517$ ,  
 $F^{(4)} = 42471$ ,  $\zeta = 370$ ,  $B_{20} = 8444.4$ , and  $B_{40} = -22975$   
 $(Dq = 1552.10, B = 610.65, \text{ and } C = 3370.71)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
15	4,5	15044	0.44	4G	+	0.28	4F	+	0.25	4D
16	4,5	15059	0.44	4G	+	0.28	4F	+	0.25	4D
17	6	15078	0.43	4G	+	0.28	4F	+	0.25	4D
18	4,5	15101	0.43	4G	+	0.28	4F	+	0.26	4D
19	4,5	20753	0.51	2I	+	0.26	2H	+	0.13	2F1
20	6	21232	0.53	2I	+	0.23	2H	+	0.11	2F1
21	4,5	21260	0.51	2I	+	0.40	2F1	+	0.06	2F2
22	4,5	21511	0.51	2I	+	0.21	2H	+	0.12	2F1
23	4,5	22090	0.29	4G	+	0.29	2I	+	0.14	2F1
24	6	22189	0.32	2I	+	0.17	4G	+	0.16	2F1
25	4,5	22434	0.72	4G	+	0.11	2I	+	0.06	2F1
26	4,5	22667	0.89	4G	+	0.03	2I	+	0.03	4F
27	6	22766	0.82	4G	+	0.06	2I	+	0.02	2H
28	4,5	22889	0.77	4G	+	0.09	2I	+	0.04	2F1
29	4,5	23073	0.88	4G	+	0.06	2I	+	0.02	2H
30	6	23075	0.91	4G	+	0.05	2I	+	0.02	2H
31	4,5	23182	0.38	2I	+	0.21	4G	+	0.18	2F1
32	4,5	24417	0.47	4G	+	0.39	4D	+	0.05	2I
33	6	24455	0.47	4G	+	0.40	4D	+	0.04	2I
34	4,5	24526	0.50	4G	+	0.44	4D	+	0.03	4F
35	4,5	24601	0.49	4G	+	0.46	4D	+	0.03	4F
36	6	25057	0.30	2I	+	0.19	2H	+	0.12	4D
37	4,5	25124	0.30	2I	+	0.21	2H	+	0.11	2D3
38	4,5	25528	0.58	4D	+	0.35	4G	+	0.02	2I
39	6	25534	0.53	4D	+	0.32	4G	+	0.06	2I
40	4,5	26818	0.92	4D	+	0.03	4G	+	0.03	4F
41	6	26839	0.92	4D	+	0.03	4G	+	0.03	4F
42	4,5	26843	0.93	4D	+	0.03	4G	+	0.03	4F
43	4,5	26854	0.93	4D	+	0.03	4G	+	0.02	4F
44	6	27926	0.31	2G2	+	0.19	2H	+	0.18	2F2
45	4,5	27969	0.29	2G2	+	0.19	2H	+	0.18	2F2
46	4,5	28473	0.22	2G2	+	0.18	2I	+	0.15	2F2
47	4,5	29641	0.34	2G2	+	0.19	2F2	+	0.18	2F1
48	6	31187	0.29	2F2	+	0.27	2G2	+	0.14	2D2
49	4,5	31216	0.27	2G2	+	0.26	2F2	+	0.15	2I
50	4,5	31493	0.34	2I	+	0.32	2G2	+	0.19	2G1
51	6	33001	0.43	4P	+	0.40	4F	+	0.07	4G
52	4,5	33024	0.46	4P	+	0.42	4F	+	0.07	4G

TABLE 20. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Tm}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 53517$ ,  
 $F^{(4)} = 42471$ ,  $\zeta = 370$ ,  $B_{20} = 8444.4$ , and  $B_{40} = -22975$   
 ( $Dq = 1552.10$ ,  $B = 610.65$ , and  $C = 3370.71$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
53	4,5	33900	0.44	2I	+	0.20	2D3	+	0.11	2D1
54	6	33958	0.41	2I	+	0.20	2D3	+	0.11	2D1
55	4,5	34795	0.58	2I	+	0.23	2G2	+	0.06	2G1
56	4,5	35320	0.43	2I	+	0.26	2D3	+	0.07	4F
57	6	35366	0.48	2I	+	0.27	2D3	+	0.06	2D1
58	4,5	35699	0.49	2I	+	0.33	2D3	+	0.06	2D1
59	4,5	35839	0.46	4F	+	0.28	4P	+	0.09	4G
60	6	35939	0.47	4F	+	0.28	4P	+	0.09	4G
61	4,5	36035	0.48	4F	+	0.29	4P	+	0.10	4G
62	4,5	36130	0.49	4F	+	0.31	4P	+	0.11	4G
63	4,5	37095	0.98	4F	+	0.01	2D3	+	0.00	2I
64	6	37106	0.98	4F	+	0.01	2D3	+	0.00	2G2
65	6	37565	0.59	2G2	+	0.12	2H	+	0.08	2I
66	4,5	37744	0.62	2G2	+	0.10	2H	+	0.09	2I
67	4,5	38853	0.34	2F2	+	0.33	2H	+	0.18	2F1
68	6	38998	0.32	2F2	+	0.24	2G2	+	0.18	2I
69	4,5	39040	0.34	2F2	+	0.22	2G2	+	0.16	2I
70	4,5	39675	0.48	2F2	+	0.27	2I	+	0.19	2G2
71	4,5	40819	0.49	2F2	+	0.32	2G2	+	0.10	2I
72	6	40990	0.39	2F2	+	0.32	2G2	+	0.11	2H
73	4,5	41108	0.44	2F2	+	0.38	2G2	+	0.05	2H
74	4,5	41923	0.29	2H	+	0.21	2F1	+	0.12	2G2
75	6	42082	0.22	2H	+	0.16	2F1	+	0.14	2G2
76	4,5	42084	0.38	4F	+	0.27	4G	+	0.23	4P
77	4,5	42209	0.34	4F	+	0.25	4G	+	0.21	4P
78	6	42294	0.29	4F	+	0.21	4G	+	0.17	4P
79	4,5	42338	0.41	4F	+	0.28	4G	+	0.24	4P
80	4,5	43356	0.53	2S	+	0.26	2G2	+	0.04	2G1
81	4,5	43495	0.39	2H	+	0.32	2F2	+	0.13	2F1
82	6	44578	0.34	2H	+	0.31	2D3	+	0.17	2D2
83	4,5	44674	0.34	2H	+	0.32	2D3	+	0.17	2D2
84	6	45612	0.46	4F	+	0.32	4G	+	0.19	4P
85	4,5	45631	0.46	4F	+	0.32	4G	+	0.19	4P
86	4,5	46010	0.32	2H	+	0.27	2F1	+	0.17	2G1
87	4,5	48099	0.61	4F	+	0.22	4D	+	0.10	4G
88	4,5	48145	0.34	4F	+	0.13	2H	+	0.12	4D
89	6	48232	0.29	4F	+	0.15	2H	+	0.14	2F1
90	4,5	48363	0.39	4F	+	0.15	4D	+	0.13	2H

TABLE 20. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Im}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F(2) = 53517$ ,  
 $F(4) = 42471$ ,  $\zeta = 370$ ,  $B_{20} = 8444.4$ , and  $B_{40} = -22975$   
 $(B_4 = 1552.10, B = 610.65, \text{ and } C = 3370.71)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	$\text{IR}^a$	Energy	Free ion state composition							
91	6	48535	0.42	4F	+	0.16	4D	+	0.09	2H
92	4,5	48615	0.50	4F	+	0.19	4D	+	0.08	4G
93	4,5	49121	0.54	4F	+	0.16	4D	+	0.12	4G
94	6	49142	0.39	4F	+	0.18	2H	+	0.11	4D
95	4,5	49354	0.28	2H	+	0.14	4F	+	0.13	2F2
96	6	49463	0.22	2H	+	0.20	4F	+	0.12	2F1
97	4,5	50130	0.30	2F1	+	0.18	2H	+	0.14	2I
98	4,5	52463	0.27	2H	+	0.23	2I	+	0.22	2F1
99	6	52536	0.27	2H	+	0.23	2I	+	0.22	2F1
100	4,5	53512	0.46	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	54237	0.53	2G1	+	0.18	2H	+	0.11	2I
102	6	54469	0.54	2G1	+	0.18	2H	+	0.11	2I
103	4,5	55741	0.36	2H	+	0.22	2I	+	0.18	2F1
104	4,5	57364	0.55	2D2	+	0.16	2G1	+	0.10	2I
105	4,5	57428	0.45	2D2	+	0.26	2G1	+	0.11	2I
106	6	57444	0.45	2D2	+	0.25	2G1	+	0.11	2I
107	4,5	58972	0.39	2G1	+	0.13	2D2	+	0.11	2H
108	6	59060	0.39	2G1	+	0.12	2D2	+	0.12	2H
109	6	61550	0.32	2D2	+	0.21	2G1	+	0.10	2I
110	4,5	61566	0.38	2D2	+	0.16	2G1	+	0.10	2F2
111	4,5	61645	0.30	2G1	+	0.24	2H	+	0.13	2D3
112	4,5	61765	0.59	2G1	+	0.13	2P	+	0.09	2F2
113	6	63008	0.20	2D3	+	0.17	2D2	+	0.16	2G1
114	4,5	63082	0.24	2G1	+	0.19	2D3	+	0.15	2H
115	4,5	63373	0.22	2D1	+	0.20	2D3	+	0.16	2G1
116	6	65077	0.48	2G1	+	0.14	2G2	+	0.10	2F2
117	4,5	65266	0.46	2G1	+	0.16	2G2	+	0.09	2D3
118	4,5	67590	0.68	2G1	+	0.16	2S	+	0.11	2G2
119	4,5	69886	0.68	2P	+	0.10	2D1	+	0.07	2F2
120	6	70024	0.68	2P	+	0.10	2D1	+	0.07	2F2
121	4,5	70186	0.78	2P	+	0.07	2G1	+	0.07	2G2
122	6	82245	0.65	2D1	+	0.10	2H	+	0.05	2G1
123	4,5	82382	0.65	2D1	+	0.10	2H	+	0.05	2F1
124	6	84260	0.59	2D1	+	0.12	2G1	+	0.11	2D3
125	4,5	84437	0.58	2D1	+	0.12	2G1	+	0.11	2D3
126	4,5	87239	0.52	2D1	+	0.14	2G1	+	0.08	2D2

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 21. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Yb}_3\text{Al}_5\text{O}_{12}$   
 [Energy levels were calculated with  $F(2) = 53780$ ,  
 $F(4) = 42292$ ,  $\zeta = 370$ ,  $B_{20} = 9544.6$ , and  $B_{40} = -23052$   
 ( $Dq = 1550.75$ ,  $B = 618.05$ , and  $C = 3356.51$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.208	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.347	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	7562	0.25	2I	+	0.23	2H	+	0.15	2F1
5	4,5	8878	0.22	2I	+	0.20	2H	+	0.14	4G
6	6	9018	0.48	4G	+	0.25	4P	+	0.07	4F
7	4,5	9254	0.56	4G	+	0.30	4P	+	0.08	4F
8	6	9491	0.33	4G	+	0.20	4P	+	0.13	2I
9	4,5	9874	0.49	4G	+	0.33	4P	+	0.06	4F
10	4,5	10001	0.42	4G	+	0.27	4P	+	0.07	2I
11	6	10024	0.32	4G	+	0.21	4P	+	0.13	2I
12	4,5	10072	0.53	4G	+	0.35	4P	+	0.05	4F
13	4,5	12513	0.48	4G	+	0.32	4F	+	0.19	4D
14	6	12540	0.48	4G	+	0.32	4F	+	0.19	4D
15	4,5	15195	0.45	4G	+	0.28	4F	+	0.24	4D
16	4,5	15214	0.44	4G	+	0.28	4F	+	0.25	4D
17	6	15239	0.44	4G	+	0.27	4F	+	0.25	4D
18	4,5	15269	0.44	4G	+	0.27	4F	+	0.25	4D
19	4,5	20564	0.51	2I	+	0.27	2H	+	0.13	2F1
20	6	21071	0.52	2I	+	0.22	2H	+	0.11	2F1
21	4,5	21142	0.51	2I	+	0.40	2F1	+	0.06	2F2
22	4,5	21309	0.48	2I	+	0.20	2H	+	0.12	2F1
23	4,5	21937	0.31	2I	+	0.25	4G	+	0.14	2F1
24	6	21977	0.33	2I	+	0.17	2F1	+	0.15	2H
25	4,5	22308	0.72	4G	+	0.10	2I	+	0.05	2F1
26	4,5	22535	0.88	4G	+	0.04	4F	+	0.03	2I
27	6	22633	0.83	4G	+	0.04	2I	+	0.03	4F
28	4,5	22775	0.74	4G	+	0.09	2I	+	0.04	2H
29	4,5	23056	0.86	4G	+	0.07	2I	+	0.03	2F1
30	6	23068	0.93	4G	+	0.03	2I	+	0.01	2H
31	4,5	23075	0.35	2I	+	0.27	4G	+	0.17	2F1
32	4,5	24371	0.49	4G	+	0.39	4D	+	0.04	2I
33	6	24406	0.49	4G	+	0.40	4D	+	0.03	4F
34	4,5	24467	0.51	4G	+	0.43	4D	+	0.03	4F
35	4,5	24537	0.51	4G	+	0.44	4D	+	0.04	4F
36	6	25089	0.31	2I	+	0.19	2H	+	0.10	2D3
37	4,5	25138	0.31	2I	+	0.21	2H	+	0.12	2D3
38	4,5	25615	0.60	4D	+	0.34	4G	+	0.02	2I

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 21. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Yb}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F^{(2)} = 53780$ ,  
 $F^{(4)} = 42292$ ,  $\zeta = 370$ ,  $B_{20} = 9544.6$ , and  $B_{40} = -23052$   
 ( $Dq = 1550.75$ ,  $B = 618.05$ , and  $C = 3356.51$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition						
39	6	25619	0.55	4D	+	0.32	4G	+	0.05 2I
40	4,5	26769	0.90	4D	+	0.04	4G	+	0.03 4F
41	6	26792	0.90	4D	+	0.04	4G	+	0.03 4F
42	4,5	26798	0.91	4D	+	0.04	4G	+	0.03 4F
43	4,5	26806	0.91	4D	+	0.04	4G	+	0.03 4F
44	6	27866	0.30	2G2	+	0.19	2H	+	0.18 2F2
45	4,5	27896	0.27	2G2	+	0.18	2H	+	0.17 2F2
46	4,5	28232	0.22	2G2	+	0.18	2I	+	0.15 2F2
47	4,5	29808	0.34	2G2	+	0.18	2F2	+	0.18 2F1
48	6	31341	0.28	2F2	+	0.27	2G2	+	0.14 2D2
49	4,5	31342	0.27	2G2	+	0.25	2F2	+	0.16 2I
50	4,5	31570	0.34	2I	+	0.31	2G2	+	0.18 2G1
51	6	32855	0.45	4P	+	0.40	4F	+	0.07 4G
52	4,5	32870	0.48	4P	+	0.41	4F	+	0.07 4G
53	4,5	33855	0.45	2I	+	0.20	2D3	+	0.11 2D1
54	6	33892	0.43	2I	+	0.20	2D3	+	0.11 2D1
55	4,5	34753	0.57	2I	+	0.23	2G2	+	0.06 2G1
56	4,5	35244	0.47	2I	+	0.29	2D3	+	0.07 2D1
57	6	35263	0.49	2I	+	0.29	2D3	+	0.07 2D1
58	4,5	35665	0.49	2I	+	0.33	2D3	+	0.06 2D1
59	4,5	36114	0.48	4F	+	0.30	4P	+	0.10 4G
60	6	36228	0.47	4F	+	0.30	4P	+	0.11 4G
61	4,5	36339	0.47	4F	+	0.31	4P	+	0.12 4G
62	4,5	36445	0.46	4F	+	0.32	4P	+	0.13 4G
63	4,5	37157	0.98	4F	+	0.01	2D3	+	0.00 2I
64	6	37166	0.97	4F	+	0.01	2G2	+	0.01 2D3
65	6	37380	0.56	2G2	+	0.14	2H	+	0.08 2I
66	4,5	37574	0.59	2G2	+	0.12	2H	+	0.08 2I
67	4,5	38520	0.34	2F2	+	0.33	2H	+	0.19 2F1
68	6	38861	0.31	2F2	+	0.24	2G2	+	0.19 2I
69	4,5	38890	0.34	2F2	+	0.21	2G2	+	0.18 2I
70	4,5	39533	0.47	2F2	+	0.29	2I	+	0.19 2G2
71	4,5	40948	0.49	2F2	+	0.32	2G2	+	0.10 2I
72	6	41041	0.39	2F2	+	0.33	2G2	+	0.10 2H
73	4,5	41156	0.44	2F2	+	0.37	2G2	+	0.05 2I
74	4,5	41926	0.31	4F	+	0.20	4G	+	0.17 4P
75	4,5	41980	0.39	4F	+	0.26	4G	+	0.21 4P
76	6	42040	0.37	4F	+	0.23	4G	+	0.20 4P

TABLE 21. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Yb}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 53780$ ,  
 $F^{(4)} = 42292$ ,  $\zeta = 370$ ,  $B_{20} = 9544.6$ , and  $B_{40} = -23052$   
 $(Dq = 1550.75, B = 618.05, \text{ and } C = 3356.51)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
77	4,5	42134	0.29	4F	+	0.20	4G	+	0.16	4P
78	4,5	42222	0.29	4F	+	0.19	4G	+	0.15	4P
79	6	42284	0.31	2H	+	0.21	2F1	+	0.15	2G2
80	4,5	43241	0.53	2S	+	0.27	2G2	+	0.04	2D2
81	4,5	43792	0.41	2H	+	0.33	2F2	+	0.12	2F1
82	6	44466	0.35	2H	+	0.30	2D3	+	0.16	2D2
83	4,5	44570	0.35	2H	+	0.31	2D3	+	0.16	2D2
84	4,5	45861	0.30	2H	+	0.29	2F1	+	0.17	2G1
85	6	45956	0.47	4F	+	0.33	4G	+	0.18	4P
86	4,5	45975	0.47	4F	+	0.33	4G	+	0.18	4P
87	4,5	48175	0.24	2H	+	0.19	2F1	+	0.15	2G1
88	6	48264	0.23	2H	+	0.19	2F1	+	0.15	2G1
89	4,5	48292	0.61	4F	+	0.23	4D	+	0.10	4G
90	4,5	48506	0.55	4F	+	0.21	4D	+	0.09	4G
91	6	48686	0.54	4F	+	0.21	4D	+	0.09	4G
92	4,5	48809	0.55	4F	+	0.22	4D	+	0.09	4G
93	4,5	49371	0.58	4F	+	0.16	4D	+	0.13	4G
94	6	49392	0.46	4F	+	0.14	2H	+	0.13	4D
95	4,5	49625	0.31	2H	+	0.15	2F2	+	0.11	4F
96	6	49716	0.27	2H	+	0.16	4F	+	0.12	2F2
97	4,5	50265	0.30	2F1	+	0.18	2H	+	0.16	2I
98	4,5	52366	0.26	2H	+	0.24	2F1	+	0.23	2I
99	6	52433	0.26	2H	+	0.23	2F1	+	0.23	2I
100	4,5	53401	0.45	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	54310	0.53	2G1	+	0.18	2H	+	0.12	2I
102	6	54558	0.53	2G1	+	0.18	2H	+	0.12	2I
103	4,5	56042	0.36	2H	+	0.22	2I	+	0.18	2F1
104	4,5	57340	0.49	2D2	+	0.24	2G1	+	0.11	2I
105	6	57380	0.48	2D2	+	0.25	2G1	+	0.11	2I
106	4,5	57433	0.55	2D2	+	0.18	2G1	+	0.09	2I
107	4,5	58948	0.41	2G1	+	0.11	2H	+	0.10	2D2
108	6	59047	0.41	2G1	+	0.12	2H	+	0.09	2D2
109	4,5	61672	0.27	2H	+	0.26	2G1	+	0.12	2D3
110	6	61950	0.33	2D2	+	0.20	2G1	+	0.10	2I
111	4,5	61956	0.40	2D2	+	0.15	2G1	+	0.11	2F2
112	4,5	62125	0.64	2G1	+	0.15	2P	+	0.11	2F2
113	6	63277	0.19	2D3	+	0.18	2G1	+	0.17	2D2
114	4,5	63320	0.24	2G1	+	0.18	2D3	+	0.12	2H

TABLE 21. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Yb}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 53780$ ,  
 $F^{(4)} = 42292$ ,  $\zeta = 370$ ,  $B_{20} = 9544.6$ , and  $B_{40} = -23052$   
 $(Dq = 1550.75, B = 618.05, \text{ and } C = 3356.51)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
115	4,5	63477	0.22	2D3	+	0.17	2D1	+	0.16	2G1
116	6	65300	0.48	2G1	+	0.14	2G2	+	0.09	2F2
117	4,5	65512	0.45	2G1	+	0.16	2G2	+	0.10	2D3
118	4,5	67829	0.67	2G1	+	0.15	2S	+	0.11	2G2
119	4,5	69898	0.65	2P	+	0.12	2D1	+	0.07	2G1
120	6	70045	0.66	2P	+	0.12	2D1	+	0.07	2F2
121	4,5	70397	0.79	2P	+	0.07	2G1	+	0.07	2G2
122	6	82339	0.65	2D1	+	0.09	2H	+	0.05	2D3
123	4,5	82490	0.65	2D1	+	0.10	2H	+	0.05	2F1
124	6	84968	0.57	2D1	+	0.12	2G1	+	0.12	2D3
125	4,5	85159	0.56	2D1	+	0.12	2G1	+	0.12	2P
126	4,5	87977	0.51	2D1	+	0.15	2G1	+	0.08	2D2

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

TABLE 22. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Lu}_3\text{Al}_5\text{O}_{12}$

[Energy levels were calculated with  $F^{(2)} = 54043$ ,  
 $F^{(4)} = 42113$ ,  $\zeta = 370$ ,  $B_{20} = 10645$ , and  $B_{40} = -23127$   
 $(Dq = 1549.34, B = 625.45, \text{ and } C = 3342.30)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
1	4,5	0.000	1.00	6S	+	0.00	4P	+	0.00	4G
2	6	0.225	1.00	6S	+	0.00	4P	+	0.00	4G
3	4,5	0.356	1.00	6S	+	0.00	4P	+	0.00	4G
4	4,5	7293	0.25	2I	+	0.23	2H	+	0.15	2F1
5	4,5	8776	0.21	2I	+	0.19	2H	+	0.16	4G
6	6	8861	0.51	4G	+	0.26	4P	+	0.08	4F
7	4,5	9074	0.56	4G	+	0.29	4P	+	0.08	4F
8	6	9381	0.32	4G	+	0.20	4P	+	0.13	2I
9	4,5	9747	0.48	4G	+	0.32	4P	+	0.06	4D
10	4,5	9874	0.40	4G	+	0.26	4P	+	0.07	2I
11	6	9916	0.29	4G	+	0.18	4P	+	0.14	2I
12	4,5	9952	0.52	4G	+	0.35	4P	+	0.06	4F
13	4,5	12280	0.49	4G	+	0.32	4F	+	0.19	4D
14	6	12309	0.49	4G	+	0.32	4F	+	0.18	4D

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.



TABLE 22. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Lu}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 54043$ ,  
 $F^{(4)} = 42113$ ,  $\zeta = 370$ ,  $B_{20} = 10645$ , and  $B_{40} = -23127$   
 (Dq = 1549.34, B = 625.45, and C = 3342.30). All  
 quantities are in  $\text{cm}^{-1}$  ]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
15	4,5	15349	0.45	4G	+	0.27	4F	+	0.24	4D
16	4,5	15373	0.45	4G	+	0.27	4F	+	0.24	4D
17	6	15403	0.45	4G	+	0.27	4F	+	0.24	4D
18	4,5	15439	0.44	4G	+	0.26	4F	+	0.25	4D
19	4,5	20364	0.50	2I	+	0.27	2H	+	0.13	2F1
20	6	20880	0.51	2I	+	0.22	2H	+	0.11	2F1
21	4,5	21010	0.49	2I	+	0.36	2F1	+	0.06	2F2
22	4,5	21074	0.47	2I	+	0.17	2H	+	0.16	2F1
23	6	21755	0.35	2I	+	0.17	2H	+	0.16	2F1
24	4,5	21782	0.34	2I	+	0.23	4G	+	0.15	2H
25	4,5	22161	0.72	4G	+	0.10	2I	+	0.05	2F1
26	4,5	22380	0.86	4G	+	0.05	4F	+	0.04	4D
27	6	22482	0.83	4G	+	0.04	4F	+	0.04	2I
28	4,5	22638	0.73	4G	+	0.09	2I	+	0.04	2H
29	4,5	22947	0.40	2I	+	0.19	2F1	+	0.17	4G
30	4,5	23051	0.95	4G	+	0.03	2I	+	0.01	2H
31	6	23062	0.95	4G	+	0.03	2I	+	0.01	2H
32	4,5	24321	0.51	4G	+	0.38	4D	+	0.03	4F
33	6	24353	0.52	4G	+	0.39	4D	+	0.03	4F
34	4,5	24408	0.53	4G	+	0.41	4D	+	0.03	4F
35	4,5	24474	0.52	4G	+	0.42	4D	+	0.04	4F
36	6	25114	0.32	2I	+	0.19	2H	+	0.11	2D3
37	4,5	25146	0.31	2I	+	0.21	2H	+	0.12	2D3
38	4,5	25704	0.62	4D	+	0.33	4G	+	0.02	2I
39	6	25706	0.57	4D	+	0.31	4G	+	0.04	2I
40	4,5	26715	0.87	4D	+	0.05	4G	+	0.03	4F
41	6	26742	0.89	4D	+	0.05	4G	+	0.03	4F
42	4,5	26750	0.90	4D	+	0.06	4G	+	0.03	4F
43	4,5	26754	0.90	4D	+	0.05	4G	+	0.03	4F
44	4,5	27779	0.23	2G2	+	0.16	2I	+	0.14	2F2
45	6	27811	0.30	2G2	+	0.20	2H	+	0.17	2F2
46	4,5	28033	0.24	2G2	+	0.16	2F2	+	0.15	2I
47	4,5	29974	0.34	2G2	+	0.18	2F2	+	0.18	2F1
48	4,5	31458	0.27	2G2	+	0.23	2F2	+	0.17	2I
49	6	31487	0.27	2G2	+	0.27	2F2	+	0.13	2D2
50	4,5	31646	0.35	2I	+	0.31	2G2	+	0.17	2G1
51	6	32708	0.47	4P	+	0.39	4F	+	0.07	4G
52	4,5	32716	0.49	4P	+	0.40	4F	+	0.07	4G

TABLE 22. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Lu}_3\text{Al}_5\text{O}_{12}$  (cont'd)  
 [Energy levels were calculated with  $F(2) = 54043$ ,  
 $F(4) = 42113$ ,  $\zeta = 370$ ,  $B_{20} = 10645$ , and  $B_{40} = -23127$   
 ( $Dq = 1549.34$ ,  $B = 625.45$ , and  $C = 3342.30$ ). All  
 quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>d</sup>	Energy	Free ion state composition							
53	4,5	33808	0.46	2I	+	0.19	2D3	+	0.10	2D1
54	6	33821	0.44	2I	+	0.20	2D3	+	0.10	2D1
55	4,5	34696	0.56	2I	+	0.22	2G2	+	0.06	2G1
56	4,5	35132	0.46	2I	+	0.29	2D3	+	0.07	2D1
57	6	35132	0.48	2I	+	0.29	2D3	+	0.07	2D1
58	4,5	35633	0.49	2I	+	0.32	2D3	+	0.06	2D1
59	4,5	36417	0.45	4F	+	0.31	4P	+	0.12	4G
60	6	36538	0.44	4F	+	0.31	4P	+	0.12	4G
61	4,5	36658	0.44	4F	+	0.32	4P	+	0.13	4G
62	4,5	36768	0.43	4F	+	0.33	4P	+	0.14	4G
63	6	37162	0.44	2G2	+	0.20	4F	+	0.13	2H
64	4,5	37218	0.97	4F	+	0.01	2G2	+	0.01	2D3
65	6	37259	0.80	4F	+	0.10	2G2	+	0.03	2H
66	4,5	37392	0.56	2G2	+	0.14	2H	+	0.07	2I
67	4,5	38181	0.34	2F2	+	0.33	2H	+	0.19	2F1
68	6	38725	0.31	2F2	+	0.23	2G2	+	0.20	2I
69	4,5	38743	0.33	2F2	+	0.21	2G2	+	0.19	2I
70	4,5	39388	0.45	2F2	+	0.31	2I	+	0.18	2G2
71	4,5	41070	0.48	2F2	+	0.31	2G2	+	0.09	2I
72	6	41092	0.38	2F2	+	0.33	2G2	+	0.09	2H
73	4,5	41201	0.42	2F2	+	0.37	2G2	+	0.05	2I
74	4,5	41844	0.41	4F	+	0.24	4G	+	0.19	4P
75	4,5	41883	0.44	4F	+	0.26	4G	+	0.21	4P
76	6	41930	0.44	4F	+	0.26	4G	+	0.21	4P
77	4,5	42034	0.42	4F	+	0.25	4G	+	0.20	4P
78	4,5	42233	0.34	2H	+	0.26	2F1	+	0.11	2G2
79	6	42342	0.32	2H	+	0.24	2F1	+	0.18	2G2
80	4,5	43112	0.52	2S	+	0.26	2G2	+	0.05	2D2
81	4,5	44081	0.44	2H	+	0.31	2F2	+	0.10	2F1
82	6	44350	0.35	2H	+	0.29	2D3	+	0.15	2D2
83	4,5	44463	0.36	2H	+	0.29	2D3	+	0.15	2D2
84	4,5	45722	0.31	2F1	+	0.26	2H	+	0.18	2F2
85	6	46309	0.48	4F	+	0.33	4G	+	0.17	4P
86	4,5	46328	0.48	4F	+	0.33	4G	+	0.18	4P
87	4,5	48135	0.29	2H	+	0.21	2F1	+	0.17	2G1
88	6	48240	0.27	2H	+	0.20	2F1	+	0.17	2G1
89	4,5	48501	0.60	4F	+	0.23	4D	+	0.10	4G
90	4,5	48705	0.59	4F	+	0.23	4D	+	0.10	4G

TABLE 22. ENERGY LEVELS OF  $\text{Fe}^{3+}$  IN  $\text{Lu}_3\text{Al}_5\text{O}_{12}$  (cont'd)

[Energy levels were calculated with  $F^{(2)} = 54043$ ,  
 $F^{(4)} = 42113$ ,  $\zeta = 370$ ,  $B_{20} = 10645$ , and  $B_{40} = -23127$   
 $(Dq = 1549.34$ ,  $B = 625.45$ , and  $C = 3342.30)$ . All  
quantities are in  $\text{cm}^{-1}$ .]

No.	IR <sup>a</sup>	Energy	Free ion state composition							
91	6	48892	0.58	4F	+	0.23	4D	+	0.10	4G
92	4,5	49038	0.56	4F	+	0.23	4D	+	0.09	4G
93	4,5	49625	0.59	4F	+	0.16	4D	+	0.14	4G
94	6	49639	0.49	4F	+	0.13	4D	+	0.12	2H
95	4,5	49890	0.34	2H	+	0.15	2F2	+	0.11	4F
96	6	49974	0.29	2H	+	0.15	4F	+	0.13	2F2
97	4,5	50379	0.31	2F1	+	0.18	2H	+	0.17	2I
98	4,5	52290	0.25	2H	+	0.25	2F1	+	0.23	2I
99	6	52350	0.25	2F1	+	0.25	2H	+	0.23	2I
100	4,5	53286	0.45	2F1	+	0.27	2I	+	0.17	2F2
101	4,5	54381	0.52	2G1	+	0.17	2H	+	0.12	2I
102	6	54643	0.53	2G1	+	0.18	2H	+	0.12	2I
103	4,5	56353	0.36	2H	+	0.21	2I	+	0.17	2F1
104	4,5	57258	0.51	2D2	+	0.23	2G1	+	0.10	2I
105	6	57317	0.51	2D2	+	0.23	2G1	+	0.10	2I
106	4,5	57534	0.55	2D2	+	0.19	2G1	+	0.08	2I
107	4,5	58959	0.44	2G1	+	0.11	2H	+	0.08	2G2
108	6	59062	0.44	2G1	+	0.12	2H	+	0.09	2G2
109	4,5	61680	0.28	2G1	+	0.26	2H	+	0.10	2I
110	4,5	62346	0.41	2D2	+	0.13	2G1	+	0.12	2F2
111	6	62358	0.33	2D2	+	0.19	2G1	+	0.10	2I
112	4,5	62507	0.63	2G1	+	0.15	2P	+	0.12	2F2
113	4,5	63502	0.21	2D3	+	0.17	2D1	+	0.15	2G1
114	6	63555	0.21	2G1	+	0.17	2D3	+	0.17	2D2
115	4,5	63677	0.26	2G1	+	0.19	2D3	+	0.13	2H
116	6	65501	0.47	2G1	+	0.14	2G2	+	0.08	2F2
117	4,5	65737	0.44	2G1	+	0.16	2G2	+	0.10	2D3
118	4,5	68080	0.67	2G1	+	0.15	2S	+	0.11	2G2
119	4,5	69915	0.62	2P	+	0.14	2D1	+	0.08	2G1
120	6	70075	0.63	2P	+	0.14	2D1	+	0.07	2G1
121	4,5	70628	0.79	2P	+	0.07	2G1	+	0.07	2G2
122	6	82452	0.65	2D1	+	0.09	2H	+	0.05	2D3
123	4,5	82616	0.66	2D1	+	0.09	2H	+	0.05	2F1
124	6	85730	0.55	2D1	+	0.13	2P	+	0.12	2G1
125	4,5	85932	0.54	2D1	+	0.14	2P	+	0.12	2G1
126	4,5	88734	0.50	2D1	+	0.16	2G1	+	0.09	2D2

<sup>a</sup>Irreducible representation of the  $C_{3i}$  group.

#### 4. DISCUSSION, CONCLUSION, AND PLANS

The final Slater parameters  $F^{(2)}$  given in table 14 shows a slight increase in going from Gd to Lu. However, the Slater parameters  $F^{(4)}$  shows a decrease over the same range. Since the shift in the Slater integrals is given [17] by  $\Delta F^{(2)} = -\rho_2^2 S^{(2)}$  and  $\Delta F^{(4)} = -\rho_4^2 S^{(4)}$  and table 13 shows that  $S^{(2)}$  and  $S^{(4)}$  increase slightly in going from Gd through Lu in the series, this would predict that  $F^{(2)}$  and  $F^{(4)}$  decrease through the series. This contradiction would indicate that either improvement in the theory or more extensive experimental data are necessary to ascertain the trend of the  $F^{(k)}$  through the series of compounds. The only crystal field parameter to change significantly is  $B_{20}$  (table 14). Since this parameter has been determined by the lattice sum parameters,  $A_{20}$ , and not by the optical spectra, this trend cannot be taken too seriously.

The purpose of tables 15 through 22 is to present realistic estimates to the spectra of  $Fe^{3+}$  in the RAG host materials for  $R = Gd$  through  $Lu$ . Hopefully, these results will aid in any further experimental efforts to measure the  $Fe^{3+}$  spectra more accurately. Further improvements in the theory will have to await better experimental results.

During the course of our work we did a preliminary investigation of  $Co^{3+}$  [18],  $Mn^{3+}$  [19], and  $Ni^{3+}$  [20], but were unable to obtain agreement of our calculation with the reported results. We cannot explain this disagreement; possibly the authors' severe round off (in the hundreds of  $cm^{-1}$ ) of their reported calculations and of their experimental data made a fitting impossible.

#### ACKNOWLEDGEMENT

The authors wish to thank summer students Ron Lee, Ken Wong, and Glen Lee for performing most of the computations. We also wish to thank Richard P. Leavitt for writing the programs used in the calculation of the energy levels.

#### LITERATURE CITED

1. V. V. Osiko, A. M. Prokhorov, and I. A. Shcherbakov, Active Media of Solid-State Lasers, *Izv. Akad. Nauk SSSR Ser. Fiz.* 44 (1980), 1698.
2. E. V. Zharikov, N. N. Il'ichev, V. V. Laptev, A. A. Malyutin, V. G. Ostroumov, P. P. Pashinin, A. S. Pimenov, V. A. Smirnov, and I. A. Shcherbakov, Spectral, Luminescence, and Lasing Properties of Gadolinium Scandium Gallium Garnet Crystals Activated with Neodymium and Chromium Ions, *Sov. J. Quantum Electron.* 13 (1983), 82.
3. A. L. Denisov, V. G. Ostroumov, Z. S. Saidov, V. A. Smirnov, and I. A. Shcherbakov, Spectral and Luminescence Properties of  $Cr^{3+}$  and  $Nd^{3+}$  Ions in Gallium Garnet Crystals, *J. Opt. Soc. Am.* B3 (1986), 95. Also, V. G. Ostroumov, Yu. S. Privis, V. A. Smirnov, and I. A. Shcherbakov, Sensitizing of  $Nd^{3+}$  Luminescence by  $Cr^{3+}$  in Gallium Garnets, *J. Opt. Soc. Am.* B3 (1986), 81.

# LITERATURE CITED (cont'd)

4. P. F. Moulton, Recent Advances in Solid State Lasers, in Digest of Conferences on Lasers and Electro-Optics '84, Optical Society of America, Washington, D.C. (1984), 76.
5. P. A. Arsenev, D. T. Sviridov, and R. K. Sviridova, Spectra of Rare Earth Metal Garnets Containing Iron Ions, Sov. Phys. Crystallogr. 17 (1972), 352.
6. International Tables for X-Ray Crystallography, Vol. I. Symmetry Groups, Kynoch, Birmingham, England, U.K. (1969).
7. C. A. Morrison and R. P. Leavitt, Handbook on the Physics and Chemistry of Rare Earths, Vol. 5, eds K. A. Gschneider and L. Eyring, North Holland, New York (1982).
8. R. P. Leavitt, On the Role of Certain Rotation Invariants in Crystal-Field Theory, J. Chem. Phys. 77 (1982), 1661.
9. P.H.M. Uylings, A.J.J. Raassen, and J. F. Wyart, Energies of N Equivalent Electrons Expressed in Terms of Two-Electron Energies and Independent Three-Electron Parameters: A New Complete Set of Orthogonal Operators: II. Application of  $3d^N$  Configurations, J. Phys. B17 (1984), 4103.
10. G. Racah, Theory of Complex Spectra IV, Phys. Rev. 76 (1949), 1352.
11. S. Fraga, L.M.S. Saxena, and J. Karwowski, Physical Science Data: 5. Handbook of Atomic Data, Elsevier, New York (1976). Also see C. A. Morrison and R. G. Schmalbach, Approximate Values of  $\langle r^k \rangle$  for the Divalent, Trivalent, and Quadrivalent Ions with the  $3d^N$  Electronic Configuration, Harry Diamond Laboratories, HDL-TL-85-3 (July 1985).
12. K. Rajnak and B. G. Wybourne, Configuration Interaction Effects in  $l^N$  Configurations, Phys. Rev. 132 (1963), 280.
13. R. E. Trees,  $4f^3$  and  $4f^2d$  Configurations of Doubly Ionized Praseodymium (PrIII), J. Opt. Soc. Am. 54 (1964), 651.
14. R. M. Macfarlane, Analysis of the Spectrum of  $d^3$  Ions in Trigonal Crystal Fields, J. Chem. Phys. 39 (1963), 3118.
15. C. W. Nielson and G. F. Koster, Spectroscopic Coefficients for  $p^n$ ,  $d^n$  and  $f^n$  Configurations, Massachusetts Institute of Technology, Cambridge, MA (1963).
16. E. Konig and S. Kremer, Ligand Field Energy Diagrams, Plenum Press, New York (1977).
17. C. A. Morrison, R. P. Leavitt, and A. F. Hansen, Host Materials for Transition-Metal Ions with the  $nd^N$  Electronic Configuration, Harry Diamond Laboratories, HDL-DS-85-1 (October 1985).

LITERATURE CITED (cont'd)

18. P. A. Arsenev and N. L. Perlova, Properties of Cobalt Ions in the Lattice of Rare Earth Aluminum Garnets, Phys. Stat. Sol. 42 (1970), K89.
19. P. A. Arsenev and V. K. Medvedeva, Mn<sup>3+</sup> Ion Properties in the Lattice of Rare Earth Aluminum Garnets, Krist. and Tech. 5 (1970), K25.
20. P. A. Arsenev, N. L. Perlova, and D. T. Sviridov, Absorption Spectra of Nickel Ions in the Crystals of Some Rare Earth Aluminum Garnets, Sov. Phys. Crystallogr. 16 (1971), 196.

## DISTRIBUTION

ADMINISTRATOR  
DEFENSE TECHNICAL INFORMATION  
CENTER  
ATTN DTIC-DDA (12 COPIES)  
CAMERON STATION, BUILDING 5  
ALEXANDRIA, VA 22314

DIRECTOR  
NIGHT VISION & ELECTRO-OPTICS  
LABORATORY  
ATTN TECHNICAL LIBRARY  
ATTN R. BUSER  
ATTN A. PINTO  
ATTN J. HABERSAT  
FT BELVOIR, VA 22060

DIRECTOR  
DEFENSE ADVANCED RESEARCH  
PROJECTS AGENCY  
ATTN J. FRIEBELE  
1400 WILSON BLVD  
ARLINGTON, VA 22209

DIRECTOR  
DEFENSE NUCLEAR AGENCY  
ATTN TECH LIBRARY  
WASHINGTON, DC 20305

UNDER SECRETARY OF DEFENSE RES  
& ENGINEERING  
ATTN TECHNICAL LIBRARY, 3C128  
WASHINGTON, DC 20301

OFFICE OF THE DEPUTY CHIEF OF STAFF,  
FOR RESEARCH, DEVELOPMENT,  
& ACQUISITION  
DEPARTMENT OF THE ARMY  
ATTN DAMA-ARZ-A, CHIEF SCIENTIST,  
L. CAMERON  
ATTN DAMA-ARZ-B, I. R. HERSHNER  
WASHINGTON, DC 20310

COMMANDER  
US ARMY ARMAMENT MUNITIONS &  
CHEMICAL COMMAND (AMCCOM)  
US ARMY ARMAMENT RESEARCH &  
DEVELOPMENT CENTER  
ATTN DRDAR-TSS, STINFO DIV  
DOVER, NJ 07801

COMMANDER  
ATMOSPHERIC SCIENCES LABORATORY  
ATTN TECHNICAL LIBRARY  
WHITE SANDS MISSILE RANGE, NM  
88002

DIRECTOR  
US ARMY BALLISTIC RESEARCH  
LABORATORY  
ATTN DRDAR-TSB-S (STINFO)  
ABERDEEN PROVING GROUND, MD  
21005

DIRECTOR  
US ARMY ELECTRONICS WARFARE  
LABORATORY  
ATTN J. CHARLTON  
ATTN DELET-DD  
FT MONMOUTH, NJ 07703

COMMANDING OFFICER  
USA FOREIGN SCIENCE &  
TECHNOLOGY CENTER  
FEDERAL OFFICE BUILDING  
ATTN DRXST-BS, BASIC SCIENCE  
DIV  
CHARLOTTESVILLE, VA 22901

COMMANDER  
US ARMY MATERIALS & MECHANICS  
RESEARCH CENTER  
ATTN DRXMR-TL, TECH LIBRARY  
WATERTOWN, MA 02172

US ARMY MATERIEL COMMAND  
5001 EISENHOWER AVE  
ALEXANDRIA, VA 22333-0001

US ARMY MATERIEL SYSTEMS  
ANALYSIS ACTIVITY  
ATTN DRXSY-MP (LIBRARY)  
ABERDEEN PROVING GROUND, MD  
21005

COMMANDER  
US ARMY MISSILE & MUNITIONS  
CENTER & SCHOOL  
ATTN ATSK-CTD-F  
ATTN DRDMI-TB, REDSTONE SCI  
INFO CENTER  
REDSTONE ARSENAL, AL 35809

COMMANDER  
US ARMY RESEARCH OFFICE  
(DURHAM)  
PO BOX 12211  
ATTN ROBERT J. LONTZ  
ATTN M. STROSIO  
ATTN M. CIFTAN  
ATTN B. D. GUENTHER

DISTRIBUTION (cont'd)

COMMANDER  
US ARMY RESEARCH OFFICE  
(DURHAM) (cont'd)  
ATTN CHARLES BOGOSIAN  
RESEARCH TRIANGLE PARK, NC  
27709

COMMANDER  
US ARMY RSCH & STD GRP (EUROPE)  
FPO NEW YORK 09510

COMMANDER  
US ARMY TEST & EVALUATION COMMAND  
ATTN D. H. SLINNEY  
ATTN TECH LIBRARY  
ABERDEEN PROVING GROUND, MD  
21005

COMMANDER  
US ARMY TROOP SUPPORT COMMAND  
ATTN DRXRES-RTL, TECH LIBRARY  
NATICK, MA 01762

COMMANDER  
ATTN DRSEL-WL-MS, ROBERT NELSON  
WHITE SANDS MISSILE RANGE, NM  
88002

OFFICE OF NAVAL RESEARCH  
ATTN J. MURDAY  
ARLINGTON, VA 22217

DIRECTOR  
NAVAL RESEARCH LABORATORY  
ATTN CODE 2620, TECH LIBRARY BR  
ATTN CODE 5554, F. BARTOLI  
ATTN CODE 5554, L. ESTEROWITZ  
ATTN CODE 5554, R. E. ALLEN  
WASHINGTON, DC 20375

COMMANDER  
NAVAL WEAPONS CENTER  
ATTN CODE 3854, R. SCHWARTZ  
ATTN CODE 3854, M. HILLS  
ATTN CODE 3844, M. NADLER  
ATTN CODE 385, R. L. ATKINS  
ATTN CODE 343, TECHNICAL  
INFORMATION DEPARTMENT  
CHINA LAKE, CA 93555

AIR FORCE OFFICE OF SCIENTIFIC  
RESEARCH  
ATTN MAJOR H. V. WINSOR, USAF  
BOLLING AFB  
WASHINGTON, DC 20332

HQ, USAF/SAMI  
WASHINGTON, DC 20330

DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS  
ATTN LIBRARY  
WASHINGTON, DC 20234

DIRECTOR  
ADVISORY GROUP ON ELECTRON DEVICES  
ATTN SECTRY, WORKING GROUP D  
201 VARICK STREET  
NEW YORK, NY 10013

AEROSPACE CORPORATION  
PO BOX 92957  
ATTN M. BIRNBAUM  
ATTN N. C. CHANG  
LOS ANGELES, CA 90009

COLORADO STATE UNIVERSITY  
PHYSICS DEPARTMENT  
ATTN S. KERN  
FORT COLLINS, CO 80523

ALLIED  
ADVANCED APPLICATION DEPT  
ATTN A. BUDGOR  
31717 LA TIEMDA DRIVE  
WESTLAKE VILLAGE, CA 91362

AMES LABORATORY DOE  
IOWA STATE UNIVERSITY  
ATTN K. A. GSCHNEIDNER, JR. (2 COPIES)  
AMES, IA 50011

ARGONNE NATIONAL LABORATORY  
ATTN W. T. CARNALL  
ATTN H. M. CROSSWHITE  
9700 SOUTH CASS AVENUE  
ARGONNE, IL 60439

ARIZONA STATE UNIVERSITY  
DEPT OF CHEMISTRY  
ATTN L. EYRING  
TEMPE, AZ 85281

BRIMROSE CORP OF AMERICA  
ATTN R. G. ROSEMEIER  
7527 BELAIR ROAD  
BALTIMORE, MD 21236

CARNEGIE MELLON UNIVERSITY  
SCHENLEY PARK  
ATTN PHYSICS & EE, J. O. ARTMAN  
PITTSBURGH, PA 15213

ENGINEERING SOCIETIES LIBRARY  
ATTN ACQUISITIONS DEPT  
345 EAST 47TH STREET  
NEW YORK, NY 10017



DISTRIBUTION (cont'd)

IBM RESEARCH DIVISION  
ALMADEN RESEARCH CENTER  
ATTN R. M. MACFARLANE  
MAIL STOP K32 802(D)  
650 HARRY ROAD  
SAN JOSE, CA 95120

JOHNS HOPKINS UNIVERSITY  
DEPT OF PHYSICS  
ATTN B. R. JUDD  
BALTIMORE, MD 21218

KALAMAZOO COLLEGE  
DEPT OF PHYSICS  
ATTN K. RAJNAK  
KALAMAZOO, MI 49007

DIRECTOR  
LAWRENCE RADIATION LABORATORY  
ATTN MARVIN J. WEBER  
ATTN HELMUT A. KOEHLER  
ATTN W. KRUPKE  
LIVERMORE, CA 94550

MARTIN MARIETTA  
ATTN F. CROWNE  
ATTN R. LEAVITT  
ATTN J. LITTLE  
ATTN T. WORCHESKY  
ATTN D. WORTMAN  
1450 SOUTH ROLLING ROAD  
BALTIMORE, MD 21227

MASSACHUSETTS INSTITUTE OF  
TECHNOLOGY  
CRYSTAL PHYSICS LABORATORY  
ATTN H. P. JENSSSEN  
ATTN A. LINZ  
CAMBRIDGE, MA 02139

MASSACHUSETTS INSTITUTE OF  
TECHNOLOGY  
77 MASS AVE  
ROOM 26-251  
ATTN V. BAGNATO  
CAMBRIDGE, MA 02139

MIT LINCOLN LAB  
PO BOX 73  
ATTN PETER MOULTON,  
ATTN B. AULL  
LEXINGTON, MA 02173

DEPARTMENT OF MECHANICAL, INDUSTRIAL,  
& AEROSPACE ENGINEERING  
PO BOX 909  
ATTN S. TEMKIN  
PISCATAWAY, NJ 08854

UNIVERSITY OF MICHIGAN  
COLLEGE OF ENGINEERING NORTH CAMPUS  
DEPARTMENT OF NUCLEAR ENGINEERING  
ATTN CHIHIRO KIKUCHI  
ANN ARBOR, MI 48104

NATIONAL OCEANIC & ATMOSPHERIC ADM  
ENVIRONMENTAL RESEARCH LABS  
ATTN LIBRARY, R-51, TECH RPTS  
BOULDER, CO 80302

OAK RIDGE NATIONAL LABORATORY  
ATTN R. G. HAIRE  
OAK RIDGE, TN 37830

OKLAHOMA STATE UNIVERSITY  
DEPT OF PHYSICS  
ATTN R. C. POWELL  
STILLWATER, OK 74078

PENNSYLVANIA STATE UNIVERSITY  
MATERIALS RESEARCH LABORATORY  
ATTN W. B. WHITE  
ATTN B. K. CHANDRASEKHAR  
UNIVERSITY PARK, PA 16802

SAN JOSE STATE UNIVERSITY  
DEPARTMENT OF PHYSICS  
ATTN J. B. GRUBER  
SAN JOSE, CA 95192

SCIENCE APPLICATIONS, INC  
ATTN T. ALLIK  
1710 GOODRIDGE DRIVE  
MCLEAN, VA 22102

SETON HALL UNIVERSITY  
CHEMISTRY DEPARTMENT  
ATTN H. BRITTAIN  
SOUTH ORANGE, NJ 07099

PRINCETON UNIVERSITY  
DEPARTMENT OF CHEMISTRY  
ATTN D. S. McCLURE  
PRINCETON, NJ 08544

UNIVERSITY OF MINNESOTA, DULUTH  
DEPARTMENT OF CHEMISTRY  
ATTN L. C. THOMPSON  
DULUTH, MN 55812

UNIVERSITY OF VIRGINIA  
DEPT OF CHEMISTRY  
ATTN DR. F. S. RICHARDSON (2 COPIES)  
ATTN DR. M. REID  
CHARLOTTESVILLE, VA 22901

# DISTRIBUTION (cont'd)

UNION CARBIDE CORP  
ATTN M. R. KOKTA  
ATTN J. H. W. LIAW  
750 SOUTH 32ND STREET  
WASHOUGAL, WA 98671

US ARMY LABORATORIES COMMAND  
ATTN TECHNICAL DIRECTOR, AMSLC-CT

INSTALLATION SUPPORT ACTIVITY  
ATTN HDL LIBRARY, SLCIS-IM-TL (3 COPIES)  
ATTN HDL LIBRARY, SLCIS-IM-TL (WOODBIDGE)  
ATTN LEGAL OFFICE, SLCIS-CC  
ATTN S. ELBAUM, SLCIS-CC

USAISC  
ATTN RECORD COPY, ASNC-ADL-TS  
ATTN TECHNICAL REPORTS BRANCH,  
ASNC-ADL-TR

HARRY DIAMOND LABORATORIES  
ATTN D/DIVISION DIRECTORS  
ATTN CHIEF, SLCHD-NW-E  
ATTN CHIEF, SLCHD-NW-EC  
ATTN CHIEF, SLCHD-NW-ED  
ATTN CHIEF, SLCHD-NW-EE  
ATTN CHIEF, SLCHD-NW-R  
ATTN CHIEF, SLCHD-NW-RA  
ATTN CHIEF, SLCHD-NW-RC  
ATTN CHIEF, SLCHD-NW-RE  
ATTN CHIEF, SLCHD-NW-RH  
ATTN CHIEF, SLCHD-NW-RI  
ATTN CHIEF, SLCHD-NW-P  
ATTN CHIEF, SLCHD-PO  
ATTN CHIEF, SLCHD-RT-C  
ATTN CHIEF, SLCHD-RT-RS  
ATTN CHIEF, SLCHD-RT-RA  
ATTN CHIEF, SLCHD-TT  
ATTN WALTER, SANDRA, SLCIS-CP-TD

HARRY DIAMOND LABORATORIES  
(cont'd)  
ATTN WILLIS, B., SLCHD-IT-EB  
ATTN ZABLUDOWSKI, B., SLCHD-IT-EB  
ATTN HERSHALL, P., SLCHD-MI-S  
ATTN KENYON, C. S., SLCHD-NW-EC  
ATTN MILETTA J. R., SLCHD-NW-EC  
ATTN McLEAN, F. B., SLCHD-NW-RC  
ATTN SATTTLER, J., SLCHD-PO-P  
ATTN LIBELO, L., SLCHD-RT-AB  
ATTN BENCIVENGA, A. A., SLCHD-RT-RC  
ATTN KULPA, S., SLCHD-RT-CB  
ATTN NEMARICH, J., SLCHD-RT-CB  
ATTN WEBER, B., SLCHD-RT-CB  
ATTN BAHDER, T., SLCHD-RT-RA  
ATTN BENCIVENGA, B. SLCHD-RT-RD  
ATTN BRODY, P., SLCHD-RT-RA  
ATTN BRUNO, J., SLCHD-RT-RA (10 COPIES)  
ATTN DROPKIN, H., SLCHD-RT-RA  
ATTN EDWARDS, SLCHD-RT-RA  
ATTN HALL, K., SLCHD-RT-RA  
ATTN HANSEN, A., SLCHD-RT-RA  
ATTN HAY, G., SLCHD-RT-RA  
ATTN KATZEN, E., SLCHD-RT-RA  
ATTN NEIFELD, R., SLCHD-RT-RA  
ATTN PENNISE, C., SLCHD-RT-RA  
ATTN SCHMALBACH, R., SLCHD-RT-RA  
ATTN SEMENDY, F., SLCHD-RT-RA  
ATTN SIMONIS, G., SLCHD-RT-RA  
ATTN SIMPSON, T., SLCHD-RT-RA  
ATTN STEAD, M., SLCHD-RT-RA  
ATTN STELLATO, J., SLCHD-RT-RA  
ATTN TOBIN, M., SLCHD-RT-RA  
ATTN TURNER, G., SLCHD-RT-RA (10 COPIES)  
ATTN WONG, B., SLCHD-RT-RA  
ATTN WORTMAN, D., SLCHD-RT-RA  
ATTN GARVIN, C., SLCHD-RT-RB  
ATTN GOFF, J., SLCHD-RT-RB  
ATTN MORRISON, C., SLCHD-RT-RA (10 COPIES)

END

9-87

Dtic